RATIONALLY INATTENTIVE HETEROGENOUS AGENTS

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Abstract

We study the effects of embedding rational inattention in a business cycle model with heterogeneous agents by comparing its predictions with the data. With a competitive labor market, rational inattention yields cross-sectional expectations accuracy for households that matches survey evidence, but fails to generate persistence in macro quantities. The opposite occurs when wages are set by households with labor market power. This conundrum arises because neither variant can simultaneously provide incentives to avoid mistakes in consumption–saving and labor supply decisions that are of similar magnitude *and* persistence in the growth rate of labor income. Moreover, conducting the same policy experiment in both variants leads to starkly different conclusions. Finally, we discuss modifications to the microfoundations, such as union-set wages and market power on the side of firms, as ways to generate robust predictions by jointly matching micro and macro evidence.

Keywords: information choice, rational inattention, business cycles, heterogenous agents, monetary policy.

JEL Codes: D83, E21, E31, E32, E71

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1. Introduction

Recent work in macroeconomics has emphasized the importance of building models that match key micro moments from the data, typically focusing on the wealth distribution and the marginal propensity to consume (MPC). This literature on household heterogeneity has reshaped our understanding of the propagation mechanisms behind aggregate fluctuations and the transmission of economic policies (see Violante 2021 for a survey).

However, most heterogeneous agents models suffer from two main weaknesses when confronted with data. First, like many of their representative agent counterparts, they usually assume perfect information, which is well known to contradict evidence from survey data showing significant belief heterogeneity among firms and households (see Carroll 2003, Weber et al. 2022). Moreover, this assumption also implies the absence of systematic correlation between individual characteristics and expectations, which runs counter to a growing body empirical evidence supporting such links (see Coibion, Gorodnichenko, and Kamdar 2018, Coibion, Gorodnichenko, and Ropele 2020). This a problematic omission given that the core philosophy of these models is to match empirical heterogeneity. Second, the presence of a realistic share of high-MPC households tends to produce shock responses that are peaked on impact, failing to generate the hump-shaped dynamics (i.e., positive autocorrelation in growth rates) for macro variables consistently documented in VAR studies (see Cogley and Nason 1995, Christiano, Eichenbaum, and Evans 2005). This is a known limitation of the framework, already noted by Kaplan, Moll, and Violante (2018).

In this paper, we start by establishing new evidence of a correlation between household heterogeneity and forecast accuracy using the *Survey of Consumer Expectations* (SCE). In particular, we find that the inflation forecast errors of hand-to-mouth households are 0.56 to 1.73 percentage points larger than those of other households. We refer to these as micro moments on expectations. Our results extend the existing evidence suggesting an interaction between households' characteristics and expectation formation (see Macaulay and Moberly 2022, Mitman et al. 2022) and shed light on a novel dimension of heterogeneity—namely, cross-sectional forecast accuracy between hand-to-mouth households and savers.[1] Given the importance of expectations in macroeconomics, neglecting this relationship between expectations and heterogeneity may lead to erroneous model-based predictions.

Next, we aim to provide an explanation for our findings using economic theory that is simultaneously consistent with macro evidence. To do so, we introduce rational inattention (Sims 1998, 2003) into a dynamic stochastic general equilibrium

[1] Mitman et al. (2022) coined this nexus the *attention channel*.

(DSGE) model with heterogeneous households. Rational inattention is a theory of information acquisition in which decision-makers allocate their attention by solving a maximization problem based on a trade-off: acquiring more precise signals about the state of the economy increases expected payoffs but also raises the attention costs associated with processing information. The combination of rational inattention and heterogeneous households yields a framework in which individual characteristics determine a household's incentives to acquire information, leading to heterogeneous signal(s) choices. Moreover, imperfect information about the state of the economy generates sluggish responses to shocks, allowing the framework to also address the lack of persistence in macroeconomic variables typically observed in this class of models. In this setting, the implied accuracy of households' forecasts and macroeconomic dynamics are jointly determined in general equilibrium.

For our analysis, we adopt a stylized view of heterogeneity based on the long tradition of two-agent models (Campbell and Mankiw 1989, Bilbiie 2008). Households are divided into constant shares of savers ($\mathcal{S}$) and hand-to-mouth ($\mathcal{H}$). This dimension of heterogeneity is sufficient to replicate most of the responses to aggregate shocks implied by state-of-the-art heterogeneous agents economies and yields an aggregate MPC on impact that is consistent with the data.[2] More importantly, the setup is tractable enough to solve for the optimal signal(s) about the state of economy of each agent type in a dynamic environment and for the economy's general equilibrium as a function of the distribution of expectations generated by those signals.

We quantitatively evaluate our model using data on macroeconomic variables and micro moments of expectations. We begin our analysis with a version of the model in which the wage rate is determined competitively. In this setting, the incentives to avoid mistakes in consumption–saving and labor-supply decisions are of similar magnitude, leading savers to pay more attention than hand-to-mouth households and to form more accurate forecasts. Savers' inflation forecasts are 0.42 percentage points more accurate than those of hand-to-mouth households, which is close to our empirical range of 0.56 to 1.73. However, rational inattention is a weak propagation mechanism with competing effects. The more inattentive savers are, the more delayed their consumption response to shocks, inducing positive autocorrelation in its growth rate. At the same time, greater inattention generates more inertia in labor-supply decisions, requiring a more volatile wage rate for the labor market to clear. This second effect induces negative autocorrelation in the consumption growth rate of hand-to-mouth households. In our calibration, these two effects lead to auto-

[2] See Debortoli and Galí 2024 for a discussion. All models considered in their analysis suffer from the two weaknesses mentioned above.

correlation in aggregate output growth of –0.06, far from the value of 0.3 observed in the data. This result contrasts with a Ricardian version of the economy, where inattention can generate any desired degree of output persistence.

Next, we evaluate a version of the model that addresses the problem of wages being counterfactually too responsive to shocks when determined competitively. We make the standard assumption that households have some market power, allowing them to set the wage for their differentiated variety of labor. This change in labor market structure resolves the problem of weak persistence in output growth—the model now matches the value observed in the data—and generates empirically plausible dynamics for the real wage. Inattention produces hump-shaped labor income responses through sluggish wage adjustments, leading to positive autocorrelation in the consumption growth of hand-to-mouth households. However, with labor market power, the consequences of setting wages suboptimally are significantly larger because firms can substitute toward labor varieties with lower wages. As a result, the incentives to avoid mistakes in wage-setting decisions greatly exceed those in consumption–saving. Since both types of households are wage-setters, they design similar signals, leading to differences in forecast accuracy between the two types that are inconsistent with the data. The direction of the accuracy difference is even reversed, with hand-to-mouth households' inflation expectations being 0.07 percentage points more accurate than those of savers.

The two variants of our model differ not only in their fit to micro and macro moments but also in their predictions regarding propagation and policy experiments. For example, under competitive wages, varying the share of hand-to-mouth households—which can be interpreted as a policy that lowers barriers to financial markets—leads to significant changes in aggregate dynamics. In the extreme case where this share is set to zero, the first-order serial correlation of output growth increases from -0.06 to 0.19 while the magnitude of output's response to an expansionary monetary policy shock declines by one-third. By contrast, in the model with market power, the effect is negligible for both outcomes.

Our results reveal a novel challenge: embedding rational inattention in a baseline business cycle model with heterogenous households does not allow us to simultaneously match the persistence of macro data and cross-sectional differences in expectations from the survey evidence. At the heart of the issue is the labor market, which must be structured to generate hump-shaped response of labor income to shocks while providing incentives to avoid mistakes in consumption-saving and labor supply decisions that are of similar magnitude. Moreover, depending on one's preferred philosophy—emphasizing the matching of micro moments, as in the heterogeneous-agent literature, or macro data, as in the representative-consumer literature—conclusions about policy experiments based on the chosen models differ starkly. For robust predictions, one would like to set up a model in which wages adjust sluggishly, inducing inertial responses of households' labor income to shocks while removing the wage-setting decision from households' attention problem, thereby preserving savers' higher incentives to pay attention. Plausible examples of such environments include model variants with union-negotiated wages subject to adjustment frictions or with monopsonistic labor markets.

Finally, we compare rational inattention with other propagation mechanisms in the literature that address either of the aforementioned weaknesses in heterogeneous-agent models (i.e., the lack of persistence in macro quantities and the assumption of perfect information). In terms of macroeconomic dynamics, our framework has the appealing property of generating inertia in quantities without requiring modifications to the functional form of preferences or affecting the MPC—unlike habit formation in consumption. In terms of expectations, rational inattention introduces information frictions that are essential to match the differences in cross-sectional forecast accuracy observed in the data. The model's predictions about whose expectations are the most accurate are endogenously determined and depend on the incentives of each type of household to pay attention. In comparison to exogenous information structures, which can always be specified to match survey data, this has two advantages: expectations change with policy and are thus not subject to the *Lucas critique*, and the role of information frictions on macro dynamics is disciplined by its implications for the cross-sectional distributions of forecasts.

Related Litterature. Maćkowiak and Wiederholt (2015, 2023) solve benchmark business cycle models with rationally inattentive Ricardian households. They show that rational inattention is a strong propagation mechanism, capable of simultaneously matching the impulse responses of macro shocks and producing empirically plausible measures of aggregate information rigidity. We show that in models where a fraction of households face borrowing constraints, achieving a good fit in terms of both macro persistence and micro moments on expectations is challenging when assuming standard microfoundations for the labor market.

Song and Stern (2020) solve a model with rationally inattentive firms that are heterogeneous in terms of attention costs. Similarly to our model, when the fraction of firms of different types changes, so do the real effects of monetary policy shocks. However, in their model, differences in attention allocation across firm types are explained exclusively by differences in information-processing capacity, not by firm characteristics. In contrast, in our setup, every household face the same marginal cost of attention, and the differences in expectations across agents are driven by heterogeneity in borrowing constraints, whose parameterization is disciplined by empirical evidence from the *Survey of Consumer Finances*.

Auclert, Rognlie, and Straub (2020) address the persistence issue in heterogeneous-agent New Keynesian models (HANK). They highlight the failure of using habit formation in consumption to generate inertial responses while preserving an empirically realistic MPC. They show that embedding sticky information *à la* Mankiw and Reis (2007) can preserve an MPC that peaks on impact and aggregate persistence. However, their proposed departure from perfect information ignores micro moments on expectations and is invariant to policy changes. As a result, fluctuations in the households' wealth distribution have no effect on the attention channel (i.e., an increase in the share of hand-to-mouth households would not reduce the average accuracy of forecasts, as suggested by our empirical evidence), and changes in policies do not affect the stickiness of information.

Gallegos (2024) studies a two-agent model with an exogenously specified imperfect information structure. In this setting, the amplification arising from high-MPC households is dampened by slow learning about fundamentals, but the model's predictions for macro dynamics are qualitatively the same as under the perfect-information benchmark. We show that information frictions—depending on the model's microfoundations—can substantially alter aggregate dynamics, almost nullifying the relevance of heterogeneity.

Mitman et al. (2022) study the interaction between heterogeneity and endogenous expectation formation in a neoclassical model *à la* Krusell and Smith (1998). They specify the set of available signals and calibrate their (monetary) costs to match data on cross-sectional expectations. In comparison, we do not target micro moments on expectations. We treat those moments as endogenous outcomes, which allows us to discriminate between different specifications of our model. In a wealth-tax experiment, Mitman et al. (2022) show that the predictions of a model with endogenous attention differ from those of a perfect-information benchmark, particularly in terms of volatility. We also find that the attention channel matters for counterfactual analysis, but in our framework its impact depends largely on the microfoundations of the labor market.

Layout. The remainder of the paper is structured as follows. Section 2 presents new empirical evidence on the relationship between hand-to-mouth status and households' forecast accuracy. Section 3 outlines the model's economic environment. Section 4 states the attention problems faced by decision-makers within the model. Section 5 defines the equilibrium of the rationally inattentive economy. Section 6 describes the dynamics and the fits of the model. Section 7 presents modifications to the microfoundations that could fit both micro and macro evidence. Section 8 compares rational inattention with other frameworks. Section 9 concludes.

2. Expectations Heterogeneity

A large body of empirical research shows that expectations are sluggish at the macro level (see Coibion and Gorodnichenko 2015, Bordalo et al. 2020) and dispersed at the micro level (see Mankiw, Reis, and Wolfers 2003, Weber et al. 2022). However, there is considerably less evidence on whether and how expectations correlate with individual characteristics.[3]

In this section, we investigate the existence of such correlation, which we refer to as *micro moments on expectations*. We do so by partitioning households into two groups, hand-to-mouth and savers, and assessing differences in average forecast accuracy between these groups. This approach is consistent with the stylized definition of heterogeneity in our theoretical framework and, to the best of our knowledge, is conducted at a broader level than comparable studies.

We use microdata from the SCE, a large panel survey held by the New York Fed that collects monthly households expectations about future macroeconomic variables such as inflation, interest rates and unemployment.[4] Typically, identifying hand-to-mouth households requires detailed microdata on finances, which the SCE lacks.[5] However, by merging the SCE with its supplemental surveys on households' spending and credit we can leverage the answers to questions on consumption behavior and liquidity constraints to assign the hand-to-mouth label to a specific set of households. Our sample spans the period from 2013M6 to 2024M4.

2.1. Measuring Expectations Relative Accuracy. We focus on forecasts of variables that are present in baseline business cycles models, namely inflation and the nominal interest rate. The SCE collects households' 12 months ahead point forecasts for inflation, and subjective probabilities of the interest rate on savings account being higher. We measure accuracy using the absolute value of forecast errors.

For inflation, measuring accuracy is straightforward, as both point forecasts and realized outcomes are observable. Our definition of inflation is the year-over-year growth rate of the Consumer Price Index (CPI), expressed in percentage points. For

[3]For existing evidence on how household and firm expectations differ both across and within groups, see Coibion, Gorodnichenko, and Kamdar (2018), Coibion, Gorodnichenko, and Ropele (2020), Reis (2020), Andrade and et al. (2022), and Macaulay and Moberly (2022).

[4]The SCE releases are available at https://www.newyorkfed.org/microeconomics/sce.

[5]Specifically, the SCE does not collect data on credit limits.

robustness, we also consider alternative measures based on the Personal Consumption Expenditures (PCE) price index and the CPI core, results are similar.

For the interest rate, the main issue is that the true underlying probability of an increase in the interest rate is unobservable.[6] To address this problem, we estimate a Bayesian Vector Autoregression (BVAR) using monthly macroeconomic data from 1960M1 to 2025M1 and infer the probability of higher interest rates from the model's posterior distribution.

2.2. Identifying Hand-to-Mouth. To identify the hand-to-mouth households in the SCE, we leverage responses to three specific questions that should systematically elicit different answers from the hand-to-mouth households compared to others (the savers).

Liquidity Constraint. In the credit survey, households are asked about their ability to respond to an unforeseen need for liquidity,

Q: *What do you think is the percent chance that you could come up with 2,000$ if an unexpected need arose within the next month?* ____

A low probability of being able to come up with the money simultaneously reflects a household's lack of liquid wealth and/or limited access to credit markets. In practice, we assigh the hand-to-mouth status to households who report a probability below 30%. A drawback of this proxy is that it may fail to capture households with zero liquid wealth who currently behave as hand-to-mouth, but would take on debt to respond to an unexpected need for liquidity.

Negative Income Shock. In the survey on spending habits, the households are asked the following multiple-choice question,

Q: *Suppose next year you were to find your household with* 10% *less household income than you currently expect. What would you do?*

1. *Cut spending by the whole amount*
2. *Not cut spending at all, but cut my savings by the whole amount*
3. *Not cut spending at all, but increase my debt by borrowing the whole amount*

[6] For unemployment, Mitman et al. (2022) approximate the true probability of higher unemployment using the average forecast from the *Survey of Professional Forecasters*, but no equivalent exists for interest rates.

4. *Cut spending by some and cut savings by some*

5. *Cut spending by some and increase debt by some.*

6. *Cut savings by some and increase debt by some.*

7. *Cut spending by some, cut savings by some and increase debt some.*

In theory, under standard assumptions on preferences featuring some form of consumption-smoothing motive, a household selecting option 1 must be hand-to-mouth, as this is the only response consistent with an inability to smooth consumption intertemporally following a negative income shock.[7] Thus, our second criterion assigns hand-to-mouth status to households that selected option 1 in response to the Negative Income Shock question. However, because the income loss is relatively large, the same caveat mentioned for the liquidity constraint proxy applies: households who behave as hand-to-mouth not due to liquidity constraints, but to avoid borrowing costs, might be missed. How relevant this concern is depends on whether those households consider a 10% income drop significant enough to take on debt. An additional source of misclassification may arise because the question does not clearly specify whether the shock is permanent or temporary; any household may choose option 1 when interpreting the income shock as permanent.

Default Probability. Lastly, in the SCE main survey, households are asked about their perceived probability of defaulting on any of their current debt,

Q: *What do you think is the percent chance that, over the next 3 months, you will NOT be able to make one of your debt payments (that is, the minimum required payments on credit and retail cards, auto loans, student loans, mortgages, or any other debt you may have)?* ____

This question can only identify a subset of hand-to-mouth households, those whom Lusardi, Schneider, and Tufano (2011) would classify as *financially fragile*. Specifically, it captures households that are near or at their borrowing limit. In practice, we classify a household as hand-to-mouth if it reports a perceived probability of defaulting greater than 70%. Results based on this proxy show if meaningful differences in the accuracy of expectations exist among hand-to-mouth households and serve as a robustness check.

[7] Interestingly, an equivalent question is asked for a hypothetical 10% positive income shock, which we expect to move households away from hand-to-mouth behavior. Indeed, nearly all respondents choose an option implying consumption smoothing.

Descriptive Statistics. For our three proxies, we compute the share of households identified as hand-to-mouth relative to the total number of households. We denote this fraction ϕ. We also compute ratios of medians and standard deviations of the absolute value of forecast errors. For each proxy, we discard outliers, defined as households whose inflation expectations are above the 95th percentile of the distribution across all households.

Table 1: Descriptive Statistics

	Negative Income Shock	Liquidity Constraint	Default Probability
ϕ (HtM share)	0.47	0.24	0.04
Inflation forecast errors			
Median ratio	0.83	0.57	0.65
S.D. ratio	0.88	0.74	0.79
Interest rate forecast errors			
Median ratio	0.95	0.85	0.97
S.D. ratio	0.98	0.97	1.03

Notes: Forecast errors measured in absolute values. Ratios are savers over hand-to-mouth.

Liquidity constraints yield the most realistic share of hand-to-mouth households compared to empirical estimates based on financial data typically reported in the litterature (see Kaplan, Violante, and Weidner 2014, Kaplan and Violante 2014). The negative income shock appears to classify too many households as hand-to-mouth, which may reflect different interpretations of the survey question by respondents. As expected, default probability captures only a small share of total households, a subset of the hand-to-mouth. Overall, Table 1 indicates that expectations of hand-to-mouth households are unconditionally less accurate and more dispersed than those of other households for both inflation and the probability of interest rates increasing.

Table 2: Median Liquid Assets

	Negative Income Shock	Liquidity Constraint	Default Probability
Hand-to-mouth	3,450$	-4,500$	-20,000$
Savers	10,500$	15,000$	8,000$

Notes: Liquid assets calculated as the current value of savings accounts (excluding retirement accounts) minus outstanding debt (excluding housing).

Table 2 presents the median value of liquid assets for the hand-to-mouth and the remaining households according to each of our alternative classification criteria. The reported values confirm that our proxies successfully identify households with lower liquid assets as hand-to-mouth.

2.3. Relative Accuracy of Hand-to-mouth Expectations. Unconditional differences in expectations across our household partitions could simply reflect composition effects. The SCE collects additional information that allows us to conduct a regression analysis controlling for individual characteristics that may influence household's information processing ability and thus its forecast quality.

Table 3 reports regressions of the absolute value of inflation forecast errors on a binary variable indicating whether a household is classified as hand-to-mouth according to each of our three criteria. Controlling for education and numeracy, hand-to-mouth households make significantly less accurate forecasts than other households.

Table 3: Relative Inflation Expectations Accuracy of Hand-to-mouth Households

	Negative Income Shock	Liquidity Constraint	Default Probability
	-	-	-
Hand-to-mouth	0.563***	1.734***	1.731***
	(0.030)	(0.038)	(0.071)
High School	-	-	-
Some College	−0.732***	−0.751***	−0.862**
	(0.038)	(0.039)	(0.035)
College	−1.848***	−1.764***	−2.178***
	(0.038)	(0.040)	(0.035)
Low Numeracy	-	-	-
High Numeracy	−2.087***	−2.061***	−2.474***
	(0.033)	(0.035)	(0.030)
Unemployed	-	-	-
Part-time employed	0.041	0.107**	−0.013
	(0.048)	(0.050)	(0.044)
Full-time employed	−0.508***	−0.399***	−0.552***
	(0.033)	(0.034)	(0.030)
Observations	109,879	112,972	156,160
F Statistic	1783.44	2306.78	2981.38
R^2	0.112	0.133	0.125
Time Fixed Effects	yes	yes	yes

Notes: Estimates from a regression of the absolute value of inflation forecast errors on the household hand-to-mouth status identified with different proxies. Estimates are relative to households that do not qualify as hand-to-mouth. Robust standard errors in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Sample: 2013M6-2024M4.

These empirical results are a first contribution of the paper; novel micro moments about expectations that can be used to evaluate or calibrate information structures in heterogeneous agent models. In the remainder of the paper, we turn to theoretical business cycle models with rationally inattentive agents, and test whether they can replicate the cross-sectional differences in expectations accuracy reported in Table 3, while also generating empirically consistent dynamics for macroeconomic variables.

3. Economic Environment

In this section, we describe the features of a baseline two-agent economy with a competitive labor market, excluding attention problems. Time is discrete, and periods correspond to quarters. Aggregate fluctuations are driven by two exogenous shocks: aggregate technology and innovations to the monetary policy rule. We focus on these shocks for two reasons. First, prior work shows that Ricardian models with inattention can generate aggregate dynamics that are consistent with the data for these shocks. Second, the presence of a demand and a supply shock ensures that households' optimal actions need not be correlated with inflation. In turn, optimal attention allocations may or may not reveal information about inflation, a micro moment we use to evaluate the model's fit.

The model features a continuum of firms producing differentiated goods, a Central Bank setting the nominal interest rate, and a continuum of households composed of savers and hand-to-mouth. This framework resembles Bilbiie (2008, 2020), and Debortoli and Galí (2024), except that all sources of New Keynesian adjustment frictions are discarded.

In Appendix E, we describe a modified version of the model in which households supply differentiated labor services and have market power to set the wage rate for their labor variety. In Section 6, we contrast the quantitative results from this monopolistic-competition labor market version of the model with those from the version featuring a competitive labor market.

3.1. Households. There is a continuum of households indexed by $j \in [0, 1]$, each of which belongs to one of two types $h \in \{\mathcal{H}, \mathcal{S}\}$. Each household keeps its type permanently and seeks to maximize its expected discounted sum of period utilities. The discount factor is $\beta \in (0, 1)$, and the period utility function is

$$U(C_{j_t}, L_{jt}) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \varphi^h \frac{L_{jt}^{1+\psi}}{1+\psi} \tag{1}$$

where

$$C_{jt} = \left(\int_0^1 C_{ijt}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}. \tag{2}$$

Here, C_{jt} is a composite consumption index, C_{ijt} is the quantity consumed of variety i, and L_{jt} denotes the labor supplied by household j. Parameter γ is the inverse of the intertemporal elasticity of substitution, φ^h scales the disutility of labor, ψ is the inverse of the Frisch elasticity, and $\theta > 1$ is the preference parameter governing the elasticity of substitution between consumption varieties. Note that the disutility of labor is scaled differently for saver and hand-to-mouth households. This is an innocuous assumption ensuring that the model's non-stochastic steady-state implies the same hours worked for both types of households.[8]

Household $j \in [0, \phi]$ is a hand-to-mouth, $h = \mathcal{H}$, and its flow budget constraint is given by

$$\int_0^1 P_{it} C_{ijt} di = W_t L_{jt} - T_t^{\mathcal{H}}. \tag{3}$$

Household $j \in [\phi, 1]$ is a saver, $h = \mathcal{S}$, and its flow budget constraint is

$$\int_0^1 P_{it} C_{ijt} di + B_{jt} = R_{t-1} B_{jt-1} + W_t L_{jt} + D_t^{\mathcal{S}} - T_t^{\mathcal{S}}. \tag{4}$$

Here P_{it} is the price of variety i, W_t is the nominal wage rate, $T_t^{\mathcal{H}}$ and $T_t^{\mathcal{S}}$ are nominal lump-sum taxes, B_{jt} is household j's nominal bond holdings, R_t is the gross nominal interest rate paid on period $t-1$ nominal bonds, and $D_t^{\mathcal{S}}$ are dividends accrued from firms ownership by the savers.

We make the assumption that $B_{jt} > 0$ always holds for any household $j \in [\phi, 1]$. This allows us to write Equation (4) in terms of logged variables[9] and also effectively rules out Ponzi schemes.

Households of the same type are ex-ante identical, but due to imperfect information, they do not make the same decisions. As detailed below, each household in the inattentive economy receives noisy signal(s) about the state of the world, and idiosyncratic noise realizations lead them to take different actions.

In each period t, every household chooses consumption vector $\{C_{ijt}\}_{i\in[0,1]}$ and supply labor L_{jt} subject to Equation (3) or Equation (4). If household j is a saver, it also chooses its nominal bonds holdings B_{jt}. All decisions are made taking as given

[8] Parameter φ^h has no influence on the dynamics of the linearized stochastic version of the model.

[9] In turn, it becomes straightforward to approximate an household utility flow in terms of log-deviations from the non-stochastic steady-state.

the exogenous shocks, the vector of prices for consumption varieties, the wage rate, the nominal interest rate and all aggregate quantities.

3.2. Firms. There is a continuum $i \in [0,1]$ of firms. Firm i produces a differentiated variety of the consumption good using the production function

$$Y_{it} = e^{a_t} e^{a_{it}} L_{it}^{\alpha}. \tag{5}$$

Here, L_{it} is the quantity of labor used by firm i for production, while a_t and a_{it} are mean-zero stochastic processes for aggregate and firm-specific productivity, respectively. The parameter $\alpha \in (0,1]$ is the elasticity of output with respect to labor.

Firm i seeks to maximize the discounted sum of its period nominal profits (or dividends) given by

$$D_{it} = (1+\tau_p) P_{it} Y_{it} - W_t L_{it} \tag{6}$$

where τ_p is a production subsidy.[10]

In each period t, firm i sets a price P_{it} for its variety and demands quantity L_{it} of effective labor taking as given exogenous shocks, the vector of prices set by other firms, aggregate demand, the wage rate, the nominal interest rate and all aggregate quantities. Each firm commits to supplying any quantity of its consumption variety demanded, at the price it sets.

3.3. Government. The government consist of a monetary and a fiscal authority. The monetary authority (Central Bank) sets the nominal rate according to a Taylor rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^*}\right)^{\phi_{y^*}}\right]^{1-\rho_r} e^{v_t} \tag{7}$$

where $\Pi_t := (P_t/P_{t-1})$ denotes the inflation rate, P_t is a price index, (Y_t/Y_t^*) is the output gap defined as the ratio between actual output and its value that would prevail under perfect information and v_t is a monetary policy shock. Variables without index refer to steady-state values. The parameters ρ_r, ϕ_π and ϕ_{y^*} control the degree of inertia and the strength of the response of the monetary policy.

The governement budget constraint in period t is

[10]Formally, firms use a stochastic discount factor that is a function of the consumption flows of households of type $\mathcal{S}$, to value profits across periods. The defintion of the stochastic discount factor's functional form can be found in Appendix D.1.

$$T_t + B_t = R_{t-1}B_{t-1} + \tau_p \int_0^1 P_{it}Y_{it}di. \tag{8}$$

Following common practice, monetary policy is active and fiscal policy is passive in the sense of Leeper (1991). To finance interest on nominal bonds and subsidy payments, the government can either collect lump-sum taxes or issue new bonds. Given the presence of hand-to-mouth households, the method of government financing matters. In the following, we will assume that lump-sum taxes for hand-to-mouth households are fixed,

$$T_t^{\mathcal{H}} = T^{\mathcal{H}} \;\; \forall t \geq 0. \tag{9}$$

This assumptions allows us to abstract from considerations regarding how the government finances its expenditures in response to shocks, and guarantees that our results are not driven by unrealistic fluctuations in real taxes.

Lastly, the production subsidy is set to correct distortions arising from market power in the non-stochastic steady-state such that

$$\tau_p = \frac{\tilde{\theta}}{\tilde{\theta} - 1} - 1 \tag{10}$$

where $\tilde{\theta} > 1$ is the model implied elasticity of substitution between varieties of goods, which is strictly smaller than the preference parameter θ whenever households are inattentive.

3.4. Shocks. There are three types of exogenous shocks: monetary policy shocks, aggregate productivity, and firm-specific productivity. Firm-specific shocks ensure enough price dispersion such that the model's elasticity of substitution, $\tilde{\theta}$, is consistent with optimal attention allocation for any attention costs.

Monetary policy shocks are Gaussian innovations to the Central Bank's Taylor rule, and are assumed *i.i.d.*. Productivity shocks follow independent stationnary Gaussian first-order autoregressive processes. The cross-sectional mean of firm-specific technology averages to zero.

The vector,

$$\boldsymbol{\varepsilon_t} = \left(\varepsilon_t^v, \varepsilon_t^a, \{\varepsilon_{it}^a\}_{i\in[0,1]}\right)' \tag{11}$$

collects all of the exogenous innovations affecting the economy at time t.

3.5. Aggregation. Aggregate composite consumption and labor supply among household types are defined as

$$C_t^{\mathcal{H}} = \int_0^{\phi} C_{jt} dj \quad , \quad C_t^{\mathcal{S}} = \int_{\phi}^{1} C_{jt} dj \tag{12}$$

and

$$L_t^{\mathcal{H}} = \int_0^{\phi} L_{jt} dj \quad , \quad L_t^{\mathcal{S}} = \int_{\phi}^{1} L_{jt} dj. \tag{13}$$

Aggregate total consumption and labor supply are defined by the weighted sums

$$C_t = \phi C_t^{\mathcal{H}} + (1-\phi) C_t^{\mathcal{S}} \quad , \quad L_t^s = \phi L_t^{\mathcal{H}} + (1-\phi) L_t^{\mathcal{S}}. \tag{14}$$

Aggregate demand for consumption variety i is defined analogously. Aggregate output, labor demand and dividends are obtained by integrating over the continuum of firms

$$Y_t = \int_0^1 Y_{it} di, \;\; L_t^d = \int_0^1 L_{it}^d di, \;\; D_t = \int_0^1 D_{it} di. \tag{15}$$

Aggregate bonds, dividends, and taxes are given by

$$B_t = (1-\phi) \int_{\phi}^{1} B_{jt} dj, \;\; D_t = (1-\phi) D_t^{S}, \;\; T_t = \phi T_t^{\mathcal{H}} + (1-\phi) T_t^{\mathcal{S}} \tag{16}$$

Lastly, we assume that the price index can always be written as

$$1 = \int_0^1 d_p \left(\frac{P_{it}}{P_t} \right) di \tag{17}$$

where d_p is some twice continuously differentiabe function. Notice that this functional nests the index that would prevail under perfect information.[11]

3.6. Notation. The relative price of consumption variety i and the relative consumption of variety i by household j are denoted

$$\hat{P}_{it} = \frac{P_{it}}{P_t}, \;\; \hat{C}_{ijt} = \frac{C_{ijt}}{C_{jt}}. \tag{18}$$

The real wage rate is given by

[11] For example, when households have perfect information, $1 = \int_0^1 \left(\frac{P_{it}}{P_t} \right)^{1-\tilde{\theta}} di$, and $\tilde{\theta} = \theta$.

$$\tilde{W}_t = \frac{W_t}{P_t}, \tag{19}$$

and aggregate real fiscal variables and dividends are given by

$$\tilde{B}_t = \frac{B_t}{P_t}, \;\; \tilde{T}_t = \frac{T_t}{P_t}, \;\; \tilde{D}_t = \frac{D_t}{P_t}. \tag{20}$$

Household real bonds holdings, real taxes and real dividends for each type of households are defined analogously.

4. Attention Problems

Decision-makers subject to rational inattention face a tradeoff between processing more information, which increases their expected payof, and paying more attention, which is cognitively costly. This section describes the maximization problem that characterizes the solution to this tradeoff.

The decision maker's problem is a linear quadratic Gaussian dynamic rational inattention problem (LQG-RI). The objective is quadratic in the state and the decision-maker's actions, the state follows linear dynamics with Gaussian innovations, and the information cost is linear in Shannon's mutual information.

In order to express the objectives from Section 3 as LQG-RI problems, we compute log-quadratic approximations of the expected discounted sum of profits and utilities around the non-stochastic steady-state. In what follows we use lowercase letters to denote log-deviations of variables from the non-stochastic steady state (i.e. $x_t = \ln(X_t) - \ln(X)$). Details about these approximations are provided below and their complete derivation for each types of agents in our model is in Appendix Appendix D.

An inattentive decision-maker, indexed by ι, maximizes its quadratic objective by choosing a costly attention strategy consisting of signal(s) about the state of the economy. Formally, the decision-maker solves:

$$\max_{\boldsymbol{\Gamma}_\iota, \boldsymbol{\Sigma}_{\nu_\iota}} \left\{ \sum_{t=0}^{\infty} \beta^t E_{\iota,-1}\left[\frac{1}{2}(\boldsymbol{x}_{\iota t} - \boldsymbol{x}^*_{\iota t})' \boldsymbol{H}_{\boldsymbol{x}_\iota} (\boldsymbol{x}_{\iota t} - \boldsymbol{x}^*_{\iota t})\right] - \lambda_\iota \sum_{t=0}^{\infty} \beta^t I(\boldsymbol{\xi}_{\iota t}; \boldsymbol{S}_{\iota t} | \mathcal{I}_{\iota t-1}) \right\} \tag{21}$$

subject to

$$\boldsymbol{x}^*_{\iota t} = \boldsymbol{G}\boldsymbol{\xi}_{\iota t} \tag{22}$$

$$\boldsymbol{\xi}_{\iota t+1} = \boldsymbol{F}\boldsymbol{\xi}_{\iota t} + \boldsymbol{\mu}_{\iota t+1} \tag{23}$$

$$\boldsymbol{x}_{\iota t} = E[\boldsymbol{x}^*_{\iota t}|\mathcal{I}_{\iota t}] \tag{24}$$

$$\mathcal{I}_{\iota t} = \mathcal{I}_{\iota,-1} \cup \{\boldsymbol{S}_{\iota 0}, ..., \boldsymbol{S}_{\iota t}\} \tag{25}$$

$$\boldsymbol{S}_{\iota t} = \boldsymbol{\Gamma_\iota \xi}_{\iota t} + \boldsymbol{\nu}_{\iota t} \tag{26}$$

$$I(\boldsymbol{\xi}_{\iota t}; \boldsymbol{S}_{\iota t}|\mathcal{I}_{\iota t-1}) = H(\boldsymbol{\xi}_{\iota t}|\mathcal{I}_{\iota t-1}) - H(\boldsymbol{\xi}_{\iota t}|\mathcal{I}_{\iota t}). \tag{27}$$

with

$$\mathcal{I}_{\iota-1} \mid \boldsymbol{\Gamma_\iota}, \boldsymbol{\Sigma_{\nu_\iota}} \tag{28}$$

The vector $\boldsymbol{x}_{\iota t}$ contains the decision-maker's actions and the vector $\boldsymbol{x}^*_{\iota t}$ contains the actions they would take under perfect information. The first term appearing in Equation (21) is the part of per-period payoff (i.e. expected losses incurred from sub-optimal actions) that is affected by the decision-maker actions and has a quadratic form with weighting matrix $\boldsymbol{H_{x_\iota}}$. The second term in Equation (21) is a known quantity representing the discounted sum of information costs. The per-period information cost consists of the product between the marginal cost of attention, λ_ι, and the per-period information flow.

Equation (22) defines a linear mapping between the current state, $\boldsymbol{\xi}_{\iota t}$, and the vector of optimal actions. Given that structural shocks are Gaussian by assumption, we know that there exists at least one representation for the state vector for which this equality holds exactly.[12] Equation (23) is the law of motion of the state vector, where $\boldsymbol{\mu}_{\iota t+1}$ follows a white noise vector process with covariance matrix $\boldsymbol{\Sigma_{\mu_\iota}}$ and $\boldsymbol{F}$ is matrix with eigenvalues that may lie on the unit circle. Thus, the state-space for $\boldsymbol{\xi}_{\iota t}$ and $\boldsymbol{x}^*_{\iota t}$, described by Equation (22) and Equation (23), is linear with Gaussian innovations, but stationnarity is not imposed. [13]

The decision-maker's information set in the current period is decscribed by Equation (25). It consists of the initial information, $\mathcal{I}_{\iota,-1}$, and all signals received up to and including the current period. Equation (26) describes the signal received in period t. This equation posits that the signal loads on the period t state vector

[12] However, the relevant state vector in the presence of information frictions may be infinite-dimensional, and solving this problem numerically requires some level of approximation.

[13] For the maximization problem to be well-defined, all we need are conditional second moments to be finite which does not require stationnarity. We allow for unit roots dynamics.

according to the matrix $\boldsymbol{\Gamma}_{\boldsymbol{\iota}}$ plus $\boldsymbol{\nu}_{\iota t}$, a white noise vector process with diagonal covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\nu}_{\iota}}$. [14] Equation (27) measures the per-period information flow as the difference in the conditional entropy, $H(\cdot \mid \cdot)$, about the state before and after observing the signal in period t. This quantity represents Shannon's mutual information and quantifies the per-period uncertainty reduction. Finally, Equation (24) describes how the decision-maker optimally selects $\boldsymbol{x}_{\iota t}$ according to his information set. Given the linear Gaussian structure, the decision-maker applies the Kalman filter to optimally infer $\boldsymbol{x}^*_{\iota t}$ from any sequence of noisy signals.

Equation (28) states that the initial information set is not entierly exogenous, but at least some of its characteristics depend on the attention strategy chosen by the decision-maker. The exact relationship is described below.

The decision-maker optimizes freely over the matrices $\boldsymbol{\Gamma}_{\boldsymbol{\iota}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\nu}_{\iota}}$ (i.e., the rank of these matrices which determines the total number of signals received is endogenous) to maximize the difference between the expected discounted sum of per-period payoffs and information costs. The fundamental tradeoff is that receiving more informative signals raises both the expected payoff and information costs. All decisions regarding the attention strategy that maximize Equation (21) are made in period -1, with Equations (24) to (28) taken as given.

We make the standard assumption that the initial information set, $\mathcal{I}_{\iota,-1}$, depends on the chosen matrices $\boldsymbol{\Gamma}_{\boldsymbol{\iota}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\nu}_{\iota}}$. Specifically, given its chosen attention strategy, the decision-maker receives a long sequence of signals that places him at the steady-state of the Kalman filter in period -1. This assumption ensures that the problems of choosing $\{\boldsymbol{\Gamma}_{\iota t}, \boldsymbol{\Sigma}_{\boldsymbol{\nu}_{\iota t}}\}_{t=0}^{\infty}$ sequentially or $\boldsymbol{\Gamma}_{\boldsymbol{\iota}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\nu}_{\iota}}$ once and for all in period -1 are equivalent. All noise terms in the signals are idiosyncratic, meaning that realizations of $\boldsymbol{\nu}_{\iota t}$ are independent across firms and households, and sum to zero in the cross-section. We use this property when aggregating individual decisions.

In the following subsections we present and describe the problems of each agent in the inattentive economy. To keep things brief, we abstract from cross-sectional efficiency decisions (i.e., households' consumption mix), as these can be shown to be optimally independent from those affecting aggregate dynamics. Lastly, we assume that paying attention to aggregate and idiosyncratic shocks are separate activities. Our results are independent of this assumption, but it is practical as it allows us to remain agnostic about the stochastic process governing firm-specific productivity.

4.1. Firms. The step-by-step approximation to the expected discounted sum of firm $i \in [0, 1]$'s period profits can be found in Appendix D.1. The resulting matrix

[14] It can be shown that a signal loading on the current state plus a vector of $i.i.d.$ Gaussian noise, as described in Equation (26), is of the optimal form given the decision-maker problem defined by Equations (21) to (28). For a formal proof, see Maćkowiak, Matějka, and Wiederholt (2018).

featured in firm i's period payoff is

$$\boldsymbol{H}_{\boldsymbol{x}_i} = -(C^{\mathcal{S}})^{-\gamma} Y \left[\frac{\tilde{\theta}(\tilde{\theta}+\alpha(1-\tilde{\theta}))}{\alpha}\right]. \tag{29}$$

Firm i's vector of choice variables is

$$\boldsymbol{x}_{it} = \left(p_{it}\right)' \tag{30}$$

and its vector of optimal actions in response to aggregate shocks is

$$\boldsymbol{x}^*_{it} = \left(p_t + \frac{\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}} c_t + \frac{1}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}} \tilde{w}_t - \frac{\frac{1}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}} a_t\right)'. \tag{31}$$

4.2. Hand-to-mouth. The step-by-step approximation to the expected discounted sum of hand-to-mouth household $j \in [0, \phi]$'s period utilities can be found in Appendix D.2. The resulting matrix featured in household j's period payoff is

$$\boldsymbol{H}^{\mathcal{H}}_{\boldsymbol{x}_j} = -(C^{\mathcal{H}})^{1-\gamma} \left[\omega^{\mathcal{H}}_W(\omega^{\mathcal{H}}_W \gamma + \psi)\right]. \tag{32}$$

Household j's vector of choice variables is

$$\boldsymbol{x}_{jt} = \left(l_{jt}\right)', \tag{33}$$

its vector of optimal actions is

$$\boldsymbol{x}^*_{jt} = \left(\frac{\tilde{w}_t - \gamma c^*_{jt}}{\psi}\right), \tag{34}$$

with

$$c^*_{jt} = \omega^{\mathcal{H}}_w(\tilde{w}_t + l^*_{jt}) \tag{35}$$

4.3. Savers. The step-by-step approximation to the expected discounted sum of saver $j \in [\phi, 1]$'s period utilities can be found in Appendix D.3. An additional step is taken to obtain an expression that reduces to a pure tracking problem, with details provided in Appendix D.4. [15] The resulting matrix featured in household j's period payoff is

$$\boldsymbol{H}^{\mathcal{S}}_{\boldsymbol{x}_j} = -(C^{\mathcal{S}})^{1-\gamma} \begin{bmatrix} \gamma & 0 \\ 0 & \omega^{\mathcal{S}}_w \psi \end{bmatrix}. \tag{36}$$

[15] In a pure tracking problem, the objective is independent of the relationship between today's and tomorrow's mistakes. In other words, the state is purely exogenous, and all that matters for the decision-maker is to keep $\boldsymbol{x}_{jt}$ as close as possible to $\boldsymbol{x}^*_{jt}$.

Household j's vector of choice variables is

$$\boldsymbol{x}_{jt} = \begin{pmatrix} c_{jt}, & l_{jt} \end{pmatrix}' \tag{37}$$

and its vector of optimal actions is

$$\boldsymbol{x}^*_{jt} = \begin{pmatrix} E_{jt}\left[-\frac{1}{\gamma}(r_t - \pi_{t+1}) + c^*_{jt+1}\right] \\ \frac{\tilde{w}_t - \gamma c^*_{jt}}{\psi} \end{pmatrix} \tag{38}$$

with c^*_{jt} and c^*_{jt+1} defined accordingly to Equation (105).

The change of variables in Appendix D.4 reformulates the savers' problem in terms of consumption and labor decisions. This transformation eliminates the intertemporal interaction in the loss function that arises when households instead choose their labor supply and bond holdings, as in Appendix D.3. The intuition why errors today do not carry over in subsequent periods under this formulation is that households can always adjust their labor supply to satisfy the intratemporal optimality condition for any chosen level of consumption.

5. Equilibrium under Rational Inattention

In this section, we define an equilibrium of the inattentive economy, consisting of the economic environment from Section 3 and the attention problems from Section 4.[16] Lowercase letters denote log deviations.

Definition 1. *Given initial bonds holdings $b_{j,-1} = 0$ for every household $j \in [0, \phi]$, and for any sequence of exogenous innovations $\{\boldsymbol{\varepsilon}_t\}_{t=0}^{\infty}$, sequence of noise $\{\boldsymbol{\nu}_{it}\}_{t=0}^{\infty}$ for every firm $i \in [0, 1]$ and sequence of noise $\{\boldsymbol{\nu}_{jt}\}_{t=0}^{\infty}$ for every household $j \in [0, 1]$ a Sequential Rational Inattention Competitive Equilibrium (SRICE) is:*

- *An allocation, $\Omega_i := \left\{\boldsymbol{S}_{it}, y_{it}, l^d_{it}, p_{it}\right\}_{t=0}^{\infty}$ for every firm $i \in [0, 1]$.*
- *An allocation, $\Omega^{\mathcal{H}}_j := \left\{\boldsymbol{S}_{jt}, c_{jt}, l^s_{jt}, \{\hat{c}_{ijt}\}_{i\in[0,1]}\right\}_{t=0}^{\infty}$ for every hand-to-mouth $j \in [0, \phi]$.*
- *An allocation, $\Omega^{\mathcal{S}}_j := \left\{\boldsymbol{S}_{jt}, c_{jt}, l^s_{jt}, b^d_{jt}, \{\hat{c}_{ijt}\}_{i\in[0,1]}\right\}_{t=0}^{\infty}$ for every saver $j \in [\phi, 1]$.*
- *An allocation, $\Omega_G := \{b^s_t, t_t\}_{t=0}^{\infty}$ for the governement.*

[16] More precisely, we define an equilibrium for a linear approximation of the economy's dynamics around the nonstochastic steady state. We cannot define a global equilibrium because the attention problems in Section 4 are only valid under linear Gaussian dynamics.

- *A set of prices* $\{p_t, w_t, r_t\}_{t=0}^{\infty}$.
- *Stationnary distributions of households and firms over idiosyncratic innovations and noise.*

such that

1. *Given* $\left\{\Omega^{\mathcal{H}}_{j\in[0,\phi]}, \Omega^{\mathcal{S}}_{j\in[\phi,1]}, p_t, r_t, w_t\right\}_{t=0}^{\infty}$*, a firm's allocation,* Ω_i*, solves Section 4.1's attention problem* $\forall i \in [0,1]$.
2. *Given* $\left\{\Omega_{i\in[0,1]}, \Omega^{\mathcal{S}}_{j\in[\phi,1]}, p_t, r_t, w_t\right\}_{t=0}^{\infty}$*, a hand-to-mouth's allocation,* $\Omega^{\mathcal{H}}_j$*, solves Section 4.2's attention problem* $\forall j \in [0,\phi]$.
3. *Given* $\left\{\Omega_{i\in[0,1]}, \Omega^{\mathcal{H}}_{j\in[0,\phi]}, p_t, r_t, w_t\right\}_{t=0}^{\infty}$*, a saver's allocation,* $\Omega^{\mathcal{S}}_j$*, solves Section 4.3's attention problem* $\forall j \in [\phi,1]$.
4. *Monetary policy satisfies the Taylor rule, fiscal policy satisfies its specified rules, and given interest and subsidies payments, the governement budget is satisfied.*
5. *All markets clear* $\forall t \geq 0$ *:*
   (a) $y_t = c_t$
   (b) $l^s_t = l^d_t$
   (c) $b^s_t = b^d_t$
   (d) $y_{it} = c_{it}\ \forall i \in [0,1]$
6. *Aggregate price index is given by* $\forall t \geq 0$ *:*
   (a) $p_t = \int_0^1 p_{it} di$
7. *Aggregate quantities are given by* $\forall t \geq 0$ *:*
   (a) $y_t = \int_0^1 y_{it} di$
   (b) $c_t = \phi(C^{\mathcal{H}}/C)\int_0^{\phi} c_{jt} dj + (1-\phi)(C^{\mathcal{S}}/C)\int_{\phi}^1 c_{jt} dj$
   (c) $l^s_t = \phi(L^{\mathcal{H}}/L)\int_0^{\phi} l^s_{jt} dj + (1-\phi)(L^{\mathcal{S}}/L)\int_{\phi}^1 l^s_{jt} dj$
   (d) $l^d_t = \int_0^1 l^d_{it} di$
   (e) $b^s_t = (1-\phi)(B^{\mathcal{S}}/B)\int_{\phi}^1 b^d_{jt} dj$
   (f) $c_{it} = \phi(C^{\mathcal{H}}_i)\int_0^{\phi} c_{ijt} dj + (1-\phi)(C^{\mathcal{S}}_i)\int_{\phi}^1 c_{ijt} dj$

5.1. Computing the Aggregate Equilibrium. We now describe an iterative procedure for solving the SRICE numerically. The SRICE is a fixed point of the system of optimality and general equilibrium conditions specified in Definition 1. Compared to competitive equilibria in standard models with rational expectations, the SRICE requires expectations to be not only consistent with the aggregate law of motion of the state variables, but also that each decision maker's signals be optimal given the signals chosen by other decision makers.

An equilibrium (of aggregate prices and quantities) in the inattentive economy can always be summarized by a finite set of stochastic processes for exogenous and endogenous variables. In this case, the dynamics of the price level, the real wage, and the consumption of both household types, along with the exogenous processes for aggregate technology and monetary policy shocks, are sufficient to determine the responses of all other aggregate variables. We can solve for the endogenous processes with arbitrary precision as follows;

Algorithm. First, we guess the impulse responses over T periods for p_t, $\tilde{w}_t$, $c_t^{\mathcal{H}}$, and $c_t^{\mathcal{S}}$. This is equivalent to guessing a moving-average representation, $MA(T)$, for each of these variables.

Second, given these guesses, we compute the profit-maximizing price for a firm. We then approximate the resulting process with a finite-order VARMA[17], which yields a state-space representation of the same form as Equations (22) to (23).

Third, we solve the firm's attention problem in Section 4.1. Given the optimal signal(s), we compute the stochastic processes for the aggregate price level and labor demand implied by aggregating individual actions.

Fourth, we compute the optimal labor supply for each type of household and the optimal consumption for savers. We approximate the dynamics of utility-maximizing actions with finite VARMA processes and obtain state-space representations for each of the attention problem.

Fifth, we solve the Section 4.2 and Section 4.3's attention problems. Given the optimal signal(s), we compute the stochastic processes for consumption and labor supply for each household type implied by aggregating individual actions.

Sixth, we use the dividend equation, labor demand and labor supply to compute a process for the real wage.

Seventh, we compare the $MA(T)$ representations of the stochastic processes for p_t, $\tilde{w}_t$, $c_t^{\mathcal{H}}$, and $c_t^{\mathcal{S}}$ with their initial guesses. If any process differs by more than a prespecified tolerance, we update them via linear extrapolation, and repeat the

[17] That is, we approximate firms' optimal price decisions with a reduced-form VAR(1) with two exogenous shocks.

procedure from step two until convergence. Once convergence is achieved, we verify that labor demand equals labor supply; if so, we have found a fixed point.

When solving the attention problems, we iterate on the first-order condition using Afrouzi and Yang (2021)'s algorithm. To approximate the $MA(T)$ representations of stochastic processes as finite VARMA, we apply the frequency-domain projection method of Han, Tan, and Wu (2022). Non-stationary processes are first differenced before the projection step, and an additional row is appended to the state-space representation to restore unit root dynamics.

6. Dynamics under Rational Inattention

With the model set up and the numerical algorithm defined, we are now ready to analyze the dynamics of the inattentive economies.

6.1. Evaluation Strategy. We follow a two-step procedure. In the first step, we calibrate all structural and policy parameters except the marginal costs of attention. In the second step, we solve the model over a grid of values for marginal costs of attention. Our main objective is to isolate the role of household inattention as a propagation mechanism for the dynamic response of output to shocks. In order to discipline the attention cost parameterization we proceed as follows: for every value of households' marginal cost of attention, we choose the firm's marginal cost of attention that allows us to match the persistence of inflation in the data (0.62). We retain the value of the household inattention parameter that brings us closest to the persistence of output growth (0.3) in the data.

We quantitatively evaluate the ability of two variants of our model to generate persistence in output growth and to match the macro and micro moments discussed below. The two variants differ only in their labor market structure—one assumes perfect competition, while the other features monopolistic competition among households supplying differentiated varieties of labor.

When using data to calibrate parameters, we rely on empirical evidence from the Unites States covering the period 1969Q1–2019Q4. However, we employ the full available SCE sample to characterize micro moments on expectations as it provides more variation in inflation and other macroeconomic variables to identifying patterns in cross-sectional expectations. Restricting the analysis to the overlapping period, would not qualitatively nor significantly change our estimates.

Calibrated Parameters. The left panel of Table 4 summarizes the calibrated parameters of our model. We set the inverse of the elasticity of intertemporal substitution, γ, to 1.5 which allows for some interaction with steady-state marginal utility of consumption in the households' attention problems. We set the inverse of the Frisch elasticity, ψ, to 1.0. We assume a discount factor of $\beta = 0.99$, which corresponds to an annual return of 4% on government bonds. Aggregate technology, A, is used to normalize steady-state output to $Y = 1$. The labor share is set to $\alpha = 0.66$. The price and wage elasticity, $\tilde{\theta}$, and $\tilde{\eta}$ are set to 4, as in Maćkowiak and Wiederholt (2015).

We assume a Taylor rule with interest rate persistence, ρ_r, of 0.9, and reaction coefficients to inflation and the output gap, ϕ_π and ϕ_{y^*}, of 1.5 and 0.125, respectively.

We estimate the stochastic process for technology using the Fernald (2014) measure of Total Factor Productivity adjusted for utilization. Specifically, we linearly detrend the cumulated series and estimate a first-order autoregressive process. The resulting estimates are a persistence parameter, ρ_a, of 0.95 and a standard deviation of shocks, $100\sigma_a$, of 0.8. To estimate the standard deviation of monetary policy shocks, we invert a Taylor rule, using the parameters set above, on time series of interest rates, inflation, and the output gap over our sample. This yields a value for $100\sigma_v$ of 0.2. As long as the monetary policy shocks account for a small portion of the volatility in the nominal interest rate, this procedure should provide accurate estimates, as argued in Carvalho, Nechio, and Tristao (2021).

The share of hand-to-mouth households, ϕ, is set to 0.28, the average value our sample from the *Survey of Consumer Finances* (SCF). To obtain this average value, we estimate the fraction of hand-mouth in each available waves using Kaplan and Violante (2014)'s method. To determine the consumption ratio, $\frac{C^{\mathcal{S}}}{C^{\mathcal{H}}}$, we infer a proxy for consumption expenditure for each type of household, and take the average ratio over the sample, yielding a value of 1.4. We obtain the desired ratio by targeting the non-stochastic steady-state value of taxes $\tilde{T}^{\mathcal{H}}$. Details on the estimation of the share of hand-to-mouth and computation of the consumption proxies are provided in Appendix F.

Note that given our calibration, we do not encounter the knife-edge case where wealth and substitution effects perfectly offset each other (e.g., this occurs when $\psi = \gamma = \omega_w^{\mathcal{H}} = 1$, as in Bilbiie 2008) that would lead to a trivial attention problem for the hand-to-mouth.

6.2. Models' Fit. Table 5 presents unconditional moments from the data and the baseline models. The moments from the models are computed from the moving-average representation of the endogenous stochastic processes that define their equilibrium. Figure 1 shows the impulse response functions of output and inflation to a positive technological shock and an expansionary monetary policy shock.

We begin by commenting on the model RI-I, in which the wage rate is determined

Table 4: Calibrated and Estimated Parameters

Panel A: Calibrated Parameters

Parameter		Value
β	Discount factor	0.99
γ	EIS	1.5
ψ	Inverse Frisch	1.0
α	Labor share	0.66
$\tilde{\theta}$	Price elasticity of demand	4.0
$\tilde{\eta}$	Wage elasticity of demand	4.0
ϕ	HtM share	0.28
$\frac{C^{\mathcal{S}}}{C^{\mathcal{H}}}$	Steady-state consumption ratio	1.4
ρ_r	Taylor rule inertia	0.9
ϕ_π	Taylor rule coefficient (inflation)	1.5
ϕ_{y^*}	Taylor rule coefficient (output gap)	0.125
ρ_a	Persistence of aggregate technology	0.95
$100\sigma_a$	100× S.D. of aggregate technology shocks	0.8
$100\sigma_v$	100× S.D. of monetary policy shock	0.2

Panel B: Inattention Parameters

Parameter		Value
RI-I: competitive-wage		
λ_j	Firms marginal cost of attention	485.0
λ_j	Households marginal cost of attention	0.8
RI-II: monopolistic-wage		
λ_j	Firms marginal cost of attention	33.0
λ_j	Households marginal cost of attention	5.8
RI-F: inattentive firms, attentive households		
λ_j	Firms marginal cost of attention	360.0

Notes: Marginal attention costs scaled up by a factor of 100,000.

competitively. In terms of output growth persistence $\rho_{\Delta y}$, the model provides a poor fit. Adding inattention on the side of households is a quantitatively weak propagation mechanism, providing only a slight improvement in output growth persistence compared to the model RI-F, wherein households are perfectly attentive. We trace most of this shortcoming back to the real wage, which absorb most of the effects from shocks and is, in turn, too volatile and insufficiently persistent compared to the data.

We can decompose the mechanics leading to these dynamics and their consequences for output as follows. First, while savers adjust consumption with a delay due to inattention, increasing the persistence of output growth, they also insufficiently adjust their labor supply. Consequently, following any shock, the real wage must fluctuate even more than in a perfect-information economy, as shown by the ratio $\sigma_{\Delta w}/\sigma_{\Delta y}$ being the largest for this specification. The disposable income of hand-to-mouth agents also becomes more volatile, which has a negative effect on the autocorrelation of output growth. As a result, household inattention has a non-monotonic effect on the persistence of output growth in this model, reaching a peak at households' marginal attention costs presented in Table 4.

On the other hand, household inattention generates inflation expectations that are consistent with the data. The model-implied coefficient of hand-to-mouth status on the absolute value of one-year-ahead inflation forecast errors $\beta_{\pi,\mathcal{H}}$ is 0.42. This value is smaller than the one reported in Table 3 for our preferred proxy, but given the parsimonious economic environment, which cannot capture all the incentives households face to pay attention, we consider it meaningful. We can understand

Table 5: Unconditional Moments I

	Data	RI-I	RI-II	RI-F	PI
Targeted Moments					
ρ_π	0.62	0.62	0.62	0.62	0.023
$\rho_{\Delta y}$	0.3	-0.06	0.3	-0.14	-0.025
Untargeted Moments					
$\rho_{\Delta w}$	0.48	-0.20	0.63	-0.18	-0.025
$\sigma_\pi/\sigma_{\Delta y}$	1.06	0.33	1.15	0.37	1.17
$\sigma_{\Delta w}/\sigma_{\Delta y}$	1.10	3.72	0.80	3.07	2.81
$\beta_{\pi,\mathcal{H}}$	1.73	0.42	-0.07	-	-

Notes: Models' moments computed from the equilibrium MA representations. RI-I is the model with competitive-wage. RI-II is the model with monoplistic-wage. RI-F is the model with firms subject to inattention, and households with perfect information, and PI is the model wherein both firms and households have perfect information.

this result as follows. The competitive labor market provides little incentive for hand-to-mouth to monitor the economy, only small fluctuations in their labor supply are necessary to equalize their marginal rate of substitution to the real wage. The savers must track an additional variable: the real interest rate. This leads them to pay more attention, resulting in smaller forecasting errors compared to the hand-to-mouth households. This result remains robust to changes in preference parameters, including the inverse of the EIS and Frisch elasticity.

We now comment on the model variant with monopolistic wages, RI-II. As can be seen in the second column of Table 5, the model provides sharp improvement in terms of fitting the macro moments. Not only can it generate the desired persistence for output growth, but it also yields values for untargeted moments that are close to their empirical counterparts. In this setting, households' inattention is a strong propagation mechanism, as it also induce persistence in the growth rate of labor income, which then generates hump-shaped consumption responses for hand-to-mouth agents.

These improvements, however, come at a cost in terms of micro moments on expectations. The model-implied coefficient of hand-to-mouth status on the absolute value of one-year-ahead inflation forecast errors $\beta_{\pi,\mathcal{H}}$ is -0.07. This failure is due to the stakes associated with wage-setting decisions being so high that they constitute the main incentive for why households pay attention to the state of the economy. In this setting, it even leads hand-to-mouth agents to form expectations that are more

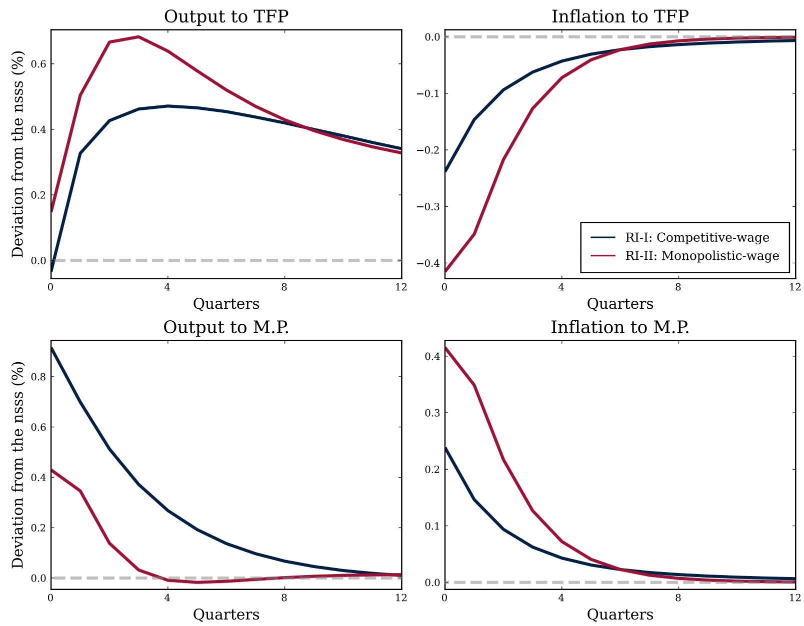

Figure 1: Impulse responses of inflation and output under rational inattention.

accurate than those of the savers, for whom the additional decisions provide only marginal incentives to pay more attention.

We now turn our attention to comparing the impulse response functions to the productivity and monetary shocks between the two variants of the model. There are three main takeaways from Figure 1. The first is that the failure of model RI-I to generate output growth persistence mainly resides in the response conditional on monetary innovations. This is to be expected, as the real rate responds to this shock monotonically and therefore induces negative autocorrelation under perfect information for the growth rate of consumption, making it harder to reverse. Secondly, for the same level of persistence, inflation is less volatile in model RI-I than in RI-II. This occurs despite the higher volatility of the real wage, as firms need to be very inattentive in this variant to match the persistence of inflation in the data, which ultimately reduces the volatility of inflation even more so than in model RI-II. Lastly, the model with competitive wage setting exhibits a weaker output response to supply shocks compared to the model with labor market power, while its response to demand shocks is relatively stronger.

6.3. The Relevance of Heterogeneity. A common exercise with heterogeneous agent models is to assess how much their dynamics differ from those of representative agent counterparts[18]. In our framework, this experiment specifically determines the extent to which the presence of hand-to-mouth households, with an MPC of one, alters the dynamics. We thus proceed to solve both baseline variants setting $\phi = 0.0$, without any changes to the other parameters. This exercise can be viewed as the extreme case of a policy that eliminates all barriers to financial access, resulting in fully Ricardian economies.

Table 6: Unconditional Moments II

		RI-I		RI-II	
	Data	TA	RA	TA	RA
ρ_π	0.62	0.62	0.75	0.62	0.69
$\rho_{\Delta y}$	0.3	-0.06	0.19	0.3	0.34
$\rho_{\Delta w}$	0.48	-0.20	-0.12	0.63	0.59
$\sigma_\pi/\sigma_{\Delta y}$	1.06	0.33	0.29	1.15	1.10
$\sigma_{\Delta w}/\sigma_{\Delta y}$	1.10	3.72	4.05	0.80	0.85

Notes: Models' moments computed from the equilibrium MA representations. TA for two-agent model. RA for Ricardian-agent model.

Table 6 presents the unconditional macro moments, and Figure 2 shows the impulse responses implied by the baseline two-agent economies and their Ricardian counterfactuals. Both confirm our previous comments on the strength and mechanisms by which households' inattention induces persistence in the growth rates of quantities. The predictions of model RI-II with monopolistic wages are barely affected when hand-to-mouth households are not present. The reason for that result is that in the variant RI-II, the consumption growth responses of each type of households are persistent. On the other hand, the Ricardian economy with competitive wages now displays positive output growth autocorrelation. This reversal occurs because labor income is irrelevant for savers' consumption decisions, and therefore the wage rate, which remains too volatile, does not hinder the capacity of inattention to induce sluggishness.

This experiment shows that for rational inattention, or any form of information friction, to act as a strong propagation mechanism in the presence of hand-to-mouth households, it must generate autocorrelation in the growth rate of labor income,

[18]An RI-DSGE is not, strictly speaking, a representative-agent model, since a continuum of households is needed for noise to sum to zero in the cross-section. Yet the average dynamics under the assumption that all households are savers are effectively the same as those implied by a representative inattentive household whose signal noise realizations are zero in every periods.

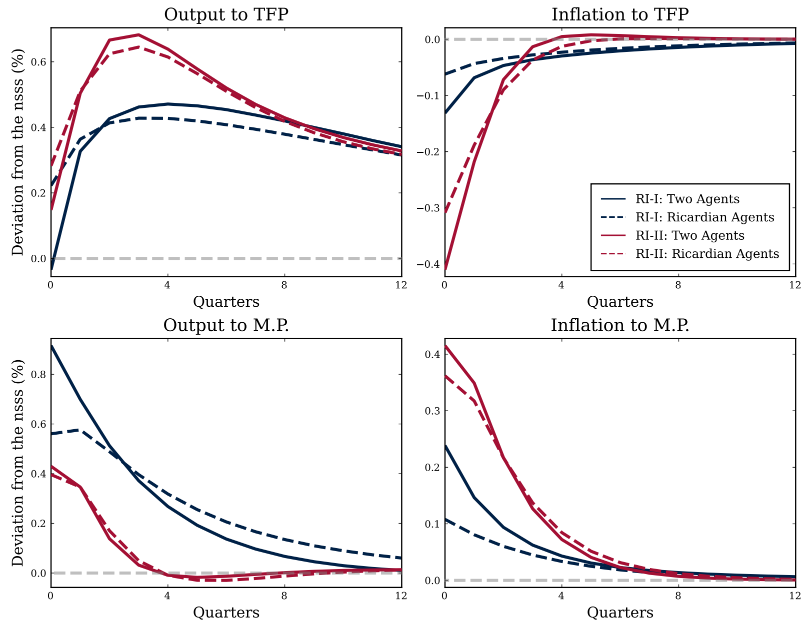

Figure 2: Impulse responses in two-agent and representative-agent economies.

which depends on the model's microfounations, namely, the labor market structure.. Assuming labor market power on the side of households achieves this goal, but it does so at the cost of producing cross-sectional expectations inconsistent with the data, casting doubt on the realism of the source of persistence in the model.

6.4. Inequality Dynamics. We conclude this section by commenting on the average dynamic responses of different variables for both types of households. Figure 3 shows the impulse response functions of consumption, hours worked, and labor income for savers and hand-to-mouth households in the two baseline economies. The first row shows the dynamics implied by a positive TFP shock, and the second row shows those implied by an expansionary monetary policy shock.

It is worthwhile to start by recalling the importance of the households' intratemporal optimality condition. In particular, together with the budget constraint, it pins down optimal hand-to-mouth consumption. Under perfect information, households make decisions such that their marginal rate of substitution equates the real wage. Under rational inattention, this condition does not hold with equality, but it still dictates the target households aim to achieve when deciding how much labor to supply or when setting wages.

Savers' consumption is always strictly determined by their beliefs about the path

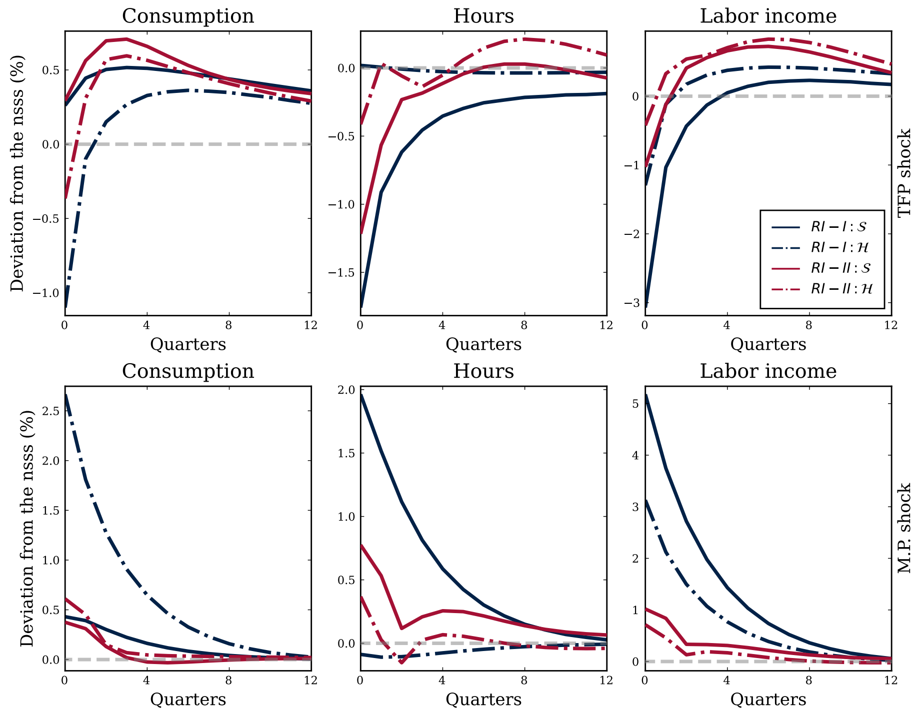

Figure 3: Impulse responses of consumption, hours, and labor income.

of the real interest rate, and, through general equilibrium effects, this affects the optimal actions of the hand-to-mouth households. In particular, the wage rate (whether set or determined in competitive equilibrium) is a function of how much labor is required to produce total output.

In the competitive-wage economy, hand-to-mouth consumption fluctuates almost one-to-one with the wage rate required to clear the labor market, and their labor supply response is almost flat. As mentioned earlier, this behavior keeps them close to equalizing their marginal rate of substitution with the real wage and is therefore near-optimal. This is not the case for savers, who should optimally also adjust labor supply because of their consumption responding to how they perceive the path of the real interest rate. As a result, hours worked and labor income are more volatile for savers than for hand-to-mouth households—an arguably unrealistic prediction given empirical evidence (see Heathcote, Perri, and Violante 2020) but unavoidable in this setting.

In this model, hand-to-mouth consumption fluctuates several percentage points more than that of savers in response to both types of shocks. In other words, the effects of labor income on hand-to-mouth consumption are larger than those of the real interest rate on savers' consumption. This pattern mirrors that of the perfect-information counterpart and implies that consumption inequality—measured as the difference between savers and hand-to-mouth households—increases following positive supply shocks but decreases in response to positive demand shocks.

On the other hand, the model where households set their wage rate predicts significant fluctuations for hours worked by hand-to-mouth households, meaning that an attention strategy implying a nearly flat labor supply curve is not attainable. Because households provide labor services that are imperfect substitutes and wages exhibit inertia due to information frictions, labor income is less volatile, and hours are distributed more evenly across types than in the RI-I model.

Hand-to-mouth consumption inherits the properties of labor income, which reduces inequality fluctuations.[19] As a result, demand shocks are less amplified than in the economy with competitive wages; the dampening of supply shocks is also mitigated. The counterfactual prediction about whose income rises most in response to shocks is likewise attenuated.

More generally, these observations illustrate why the response of output growth to a shock depends on the persistence of labor income growth (even when savers' inattention induces persistence in their own consumption). When such persistence exists, it translates into persistent hand-to-mouth consumption; when it does not, any remaining aggregate-level persistence arises despite labor income peaking on impact.

Robustness. We also solve the models with attentive firms under Calvo pricing (see Appendix G), which leaves our results unchanged. This shows that the interaction between the source of inflation persistence and household (in)attention is not critical.

7. Ways out of the Conundrum

In the previous section, we showed that baseline two-agent inattentive economies fail to simultaneously match the observed persistence in output growth (with the associated hump-shaped responses of output to shocks) *and* the greater accuracy of saver households' economic forecasts relative to hand-to-mouth households.

[19] In our calibration, this does not produce pronounced hump-shaped responses for labor income, although persistence could be increased arbitrarily by raising inattention.

We now discuss plausible modifications to the microfoundations that can resolve this conundrum.

7.1. Labor Unions. The most straightforward way out of the conundrum is to maintain the assumption that labor market power lies on the side of households, but that wage-setting decisions are delegated to representative unions subject to adjustment frictions. We use adjustment frictions as an (arguably crude) proxy for infrequent collective agreement renegotiations that are not directly carried out by households—a realistic assumption for a number of labor markets. In the present case, we assume that unions are subject to a Calvo probability of being able to reset the wage of a specific labor type, but convex adjustment costs *à la* Rotemberg would yield similar results.

In this case, then hand-to-mouth need not pay attention to the economy, while savers face a single consumption–savings decision that requires them to track the real interest rate. As a result, savers' forecast are always the most accurate, and the real wage is hump-shaped with a volatility similar to that of output growth. The relevance of heterogeneity is minimal in that setting; the consequences of setting $\phi = 0$ resemble those of the RI-II model.

This version of the model allows us to match micro and macro moments simultaneously. In particular, the source of wage stickiness—whether Calvo frictions or inattention—turns out to be largely irrelevant. Similarly, whether real wages differ across household types, as in the inattentive economy from Section 6 where each household sets its wage based on its own marginal rate of substitution, is unnecessary for generating persistence in the growth rate of labor income; assuming a single union or type-specific unions does not significantly affect the dynamics.

However, the microfoundations underlying the labor supplied by each household when unions set wages can be considered weak[20]; for details, see Appendix G. Furthermore, moving away from inattention as the source of wage inertia raises a *Lucas critique* issue that was not previously relevant: how should the frequency of wage adjustment respond to policies or changes in the economic environment?

7.2. Firms with Monopsony Power. In Section 6, we showed that cross-sectional expectations accuracy can match the data when households take the wage rate as given, because this lowers the stakes of the losses incurred due to suboptimal labor supply decisions. This, in turn, leads to a decline in the incentives to pay attention that is disproportionately larger among hand-to-mouth households relative to savers, causing them to form less accurate forecasts. An alternative way to make households wage-takers while avoiding a competitive labor market is to take inspiration from a

[20]We make the standard assumption that each household supplies a continuum of labor services.

number of recent papers (see Alpanda 2025 for a review) arguing that labor markets are often better described as monopsonistic, with market power lying with firms rather than households.

In this setting, firms set wages as a markdown relative to their marginal product of labor, while households supply labor so that their marginal rate of substitution between consumption and leisure equals the real wage. Firms being subject to inattention generates hump-shaped wage dynamics. Households taking the wage as given implies low stakes for losses incurred from suboptimal actions which leads to more accurate forecasts for the savers.

We cannot solve this model under the assumption of a purely monopsonistic labor market when firms use only labor in the production function. The reason is that the optimal price and wage rate of a firm are not individually determined in that case. We would either need to assume that, for a portion of the labor market, the power lies on the households' side, or that production requires another input such as capital. However, we conjecture that solving such a model would yield predictions consistent with both macro and micro moments in the data. Inertia in the real wage should translate into persistence in the growth rate of hand-to-mouth consumption, and, as long as a sufficient fraction of hand-to-mouth households operate in the monopsonistic labor market and take the wage rate as given, expectations should be more accurate for savers.

8. Inattention Compared to Other Frameworks

One of our main contributions is to argue and show that matching both the persistence of macro variables and the micro moments on expectations in heterogeneous agent models is not straightforward. The previous literature has typically addressed only one of these issues at a time. In this section, we compare the approaches taken in these papers to ours and discuss how they might perform if they attempted to match both sets of moments simultaneously, as we do.

8.1. Habit Formation. One common sources of slow adjustment of quantities in the representative agent literature is habit formation in consumption, as detailed in Appendix G. We find it to have similar effects as inattentive savers' consumption–savings choices, which are *essential* for the model to generate persistence in the growth rate of output even when prices and wages are sluggish. The mechanism is clearly not suited to induce differences in cross-sectional expectations. Habit formation can also be viewed as *ad-hoc*, especially in the context of baseline two-agent models, where it can only be introduced in savers' preferences, given that hand-to-mouth households solve a static maximization problem.[21] It is also hard to discipline, since the degree of habit formation may be set arbitrarily high to match desired moments irrespective of the intratemporal decision. By contrast, inertia induced by rational inattention is summarized by a single parameter that governs the speed of adjustment for all decision variables.

Moreover, habit formation has an undesirable property for the aggregate MPC in heterogeneous agents settings, as highlighted by Auclert et al. (2021). It mechanically induces slow adjustment to all shocks—both aggregate and idiosyncratic—which is not the case under inattention. For example, if households face idiosyncratic shocks that are larger than aggregate shocks, hand-to-mouth households will maintain a high MPC in response to these idiosyncratic shocks while reacting slowly to aggregate fluctuations; this arises from them endogenously allocating more attention to the idiosyncratic component.

If the goal is estimation, for instance using Bayesian techniques as in Bilbiie, Primiceri, and Tambalotti (2023), and the model is structured so that the functional form of preferences is homogeneous across types, habit formation can be employed to introduce persistence. However, it comes with all of the caveats outlined above.

8.2. Exogenous Information Structures. We have in mind frameworks that resemble Gallegos (2024) or Auclert et al. (2021). Gallegos (2024) solves a two-agent model with imperfect information, in which signals are exogenously specified as the aggregate stochastic fundamental plus a Gaussian noise term. One could also assume a more complex structure with multiple signals that are linear combinations of endogenous variables, as in Nimark (2014), but this has yet to be pursued in heterogeneous agent models. Auclert et al. (2021) assumes sticky information with respect to aggregate variables. In theory, such information structures are flexible enough to match any cross-sectional expectations patterns from the data and can induce persistence in macro variables, especially when combined with New Keynesian adjustment frictions. The contributions of imperfect information, however, can be hard to disentangle from other sources of slow adjustment in the model.

Some caveats are worth mentioning. First, in these models, the choice of signals agents receive and their precision is completely arbitrary and exogenous. While this may induce the desired moments for expectations, it is by no means evidence that these are the signals agents would choose if solving a maximization problem. Second, a *Lucas critique*–type argument arises if the model is used for counterfactual experiments. In principle, nothing guarantees that the information structure should

[21]If households can switch types, as in Bilbiie, Primiceri, and Tambalotti 2023, hand-to-mouth preferences can include habit formation.

remain unchanged following changes in policies or in the stochastic processes governing the shocks. In fact, using these environments for such counterfactual analysis completely disregards the attention channel and can lead to erroneous conclusions.

Models with exogenously fixed information structure have the advantage of being more tractable than the one we propose. If one specifies signals that are plausible, especially those that load on endogenous variables such as inflation and output, these models can still yield interesting results, particularly if they can be fully estimated.

9. Conclusions.

We solve business cycle models with heterogeneous households: hand-to-mouth consumers and savers. Models with standard labor market structures fail to simultaneously generate hump-shaped responses of macro variables and cross-sectional differences in expectation accuracy that match the data. These findings have two main implications. First, compared to non-Ricardian economies, inattention acts as a strong propagation mechanism in the presence of heterogeneous households only if it induces persistence in the growth rate of labor income. Second, inattention does not necessarily generate the same cross-sectional expectation patterns as those observed in the data. In particular, the microfoundations of the labor market, that is, the variables over which the decision makers exert influence, are crucial in this setting, as they define the incentives to acquire more or less precise signals.

We propose plausible modifications to our baseline models that allow us to obtain both the persistence in the growth of labor income and cross-sectional differences in expectation accuracy. Our results provide insights for future research on how to simultaneously introduce imperfect information and induce persistence in heterogeneous-agent models in a way that is consistent with the data. In our analysis, we consider simple economic environments that abstract from a number of sources of propagation that could be embedded in this class of models, including cyclical exposure to shocks, precautionary savings, and fiscal policy. Future work could incorporate these channels and assess whether they make it harder or easier to match the persistence of macro variables and micro moments of expectations.

Our models feature limited heterogeneity in terms of hand-to-mouth status. However, we believe that this environment still provides a reasonable approximation to more general settings in which agents can move across the wealth distribution. The reason is that under rational inattention, if individual states are sufficiently persistent (in heterogeneous-agent models, the process for idiosyncratic productivity is typically highly persistent), agents are likely to disregard the contingency associated with switching types when allocating their attention.

Finally, we want to stress that, although the models analyzed here are simpler than state-of-the-art heterogeneous-agent economies, the latter remain subject to the Lucas critique. For policy experiments—the primary use of these models—frameworks with endogenous information structures are preferable because they can yield conclusions that diverge from HANK economies due to the attention channel playing a role in shaping responses to policy experiments.

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Appendix A. Expectations Accuracy

A.1. Forecasting BVAR The model is estimated on the period 1960M1-2025M1 with Bayesian techniques (Minnesota priors on the VAR coefficients and inverse-Wishart priors on the error covariance matrix) and includes 12 lags of the CPI, Industrial Production, the Unemployment Rate, the market yield on 1-year U.S. Treasury securities, and the first three factors from McCracken and Ng (2016). We estimate the posterior distribution with 100,000 draws which we then use to forecast the interest rate one-year ahead. From these forecasts, we proxy the true probability of interest rates being higher 12M from now, which we compare with the proability reported by each households to measure expectations accuracy.

A.2. Exectations Relative Accuracy Table 7 presents the results of panel regressions on the accuracy of households' expectations and the hand-to-mouth status including the same set of controls as in Section 2 plus binary variables for the income bin to which each household belongs.

Appendix B. Perfect Information Economy

In this section, we detail the equilibrium of an economy without frictions (neither informational nor adjustment costs). The firms' and households' optimality conditions are the same that characterize their optimal actions in the inattentive economy. The non-stochastic steady state is also the same.

A perfect information economy has the following property

Definition 2. *Every decision-maker has rational expectations and knows the complete history of shocks up to, and including the current period.*

B.1. Optimality Conditions. Optimality in the frictionless perfect information economy follows from the maximization problems of firms and households. The same conditions characterize optimal actions in the inattentive economy.

Hand-to-mouth $j \in [0, \phi]$ solves a static problem that yields the following first-order condition for labor supply

$$\varphi^{\mathcal{H}} L_{jt}^{\psi} C_{jt}^{\gamma} = \tilde{W}_t. \tag{39}$$

Saver $j \in [\phi, 1]$ solves a dynamic problem, whose first-order conditions imply an Euler equation given by

$$C_{jt}^{-\gamma} = \beta E_{jt} \left[C_{jt+1}^{-\gamma} \left(\frac{R_t}{\Pi_{t+1}} \right) \right] \tag{40}$$

Table 7: Relative Expectations Accuracy of Hand-to-mouth Households

	Negative Income Shock		Liquidity Constraint		Default Probability	
	(1)	(2)	(3)	(4)	(5)	(6)
Hand-to-mouth	0.530***	0.015***	1.450***	0.021***	1.478***	−0.017***
	(0.030)	(0.001)	(0.039)	(0.002)	(0.071)	(0.003)
High School	-	-	-	-	-	-
Some College	−0.636***	−0.011***	−0.681***	−0.011***	−0.748**	−0.008**
	(0.038)	(0.002)	(0.039)	(0.002)	(0.034)	(0.001)
College	−1.484***	−0.045***	−1.450***	−0.041***	−1.772***	−0.051***
	(0.039)	(0.002)	(0.041)	(0.002)	(0.036)	(0.001)
Low Numeracy	-	-	-	-	-	-
High Numeracy	−1.861***	−0.027***	−0.021***	−2.110***	−2.237***	−0.028***
	(0.034)	(0.001)	(0.035)	(0.001)	(0.031)	(0.001)
Unemployed	-	-	-	-	-	-
Part-time employed	0.079	0.014	0.127**	−0.014***	0.034	−0.014
	(0.049)	(0.003)	(0.050)	(0.002)	(0.044)	(0.002)
Full-time employed	−0.148***	−0.002	−0.130***	−0.001	−0.188***	−0.004***
	(0.034)	(0.001)	(0.035)	(0.001)	(0.031)	(0.001)
Income < 50K	-	-	-	-	-	-
Income $\in [50, 100]$	−1.036***	−0.007***	−0.777***	−0.003**	−1.050***	−0.008***
	(0.037)	(0.002)	(0.039)	(0.001)	(0.034)	(0.001)
Income > 100K	−1.444***	−0.021***	−1.153***	−0.015**	−1.504***	−0.022***
	(0.042)	(0.002)	(0.045)	(0.002)	(0.039)	(0.002)
Observations	109,879	109,879	112,972	112,972	156,160	156,160
F Statistic	1521.92	346.71	1831.60	323.94	2471.39	529.33
R^2	0.123	0.424	0.139	0.447	0.134	0.400
Time Fixed Effects	yes	yes	yes	yes	yes	yes

Notes: Columns (1), (3), and (5) show estimates from a regression of the absolute value of inflation forecast errors on the household hand-to-mouth status identified with different proxies. Columns (2), (4), and (6) show estimates from a regression of the absolute value of the forecasts errors of the probability of interest rates going up in 12M on the household hand-to-mouth status identified with different proxies. Robust standard errors in parentheses, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Sample: 2013M6-2024M4.

and a labor supply condition

$$\varphi^{\mathcal{S}} L_{jt}^{\psi} C_{jt}^{\gamma} = \tilde{W}_t. \tag{41}$$

Firm i solves a static problem that yields the following first-order condition for optimal pricing

$$\hat{P}_{it} = \frac{\tilde{W}_t}{\alpha e^{a_t} e^{a_{it}}} \left(\frac{\hat{P}_{it}^{-\theta} C_t}{e^{a_t} e^{a_{it}}} \right)^{\frac{1-\alpha}{\alpha}}. \tag{42}$$

B.2. Non-Stochastic Steady-State. The non-stochastic steady-state is an equilibrium of the economy in the absence of shocks, with the property that real quantities, relative prices, the nominal rate, and inflation remain constant over time. In the following, variables without time-subscript denotes steady-state values.

Evaluating the first-order conditions in a non-stochastic steady-state where hours worked are symmetric across household types, $L = L^{\mathcal{H}} = L^{\mathcal{S}}$, we obtain

$$\tilde{W} = \varphi^{\mathcal{H}} L^{\psi} (C^{\mathcal{H}})^{\gamma}, \tag{43}$$

with

$$C^{\mathcal{H}} = \tilde{W} L - T^{\mathcal{H}}. \tag{44}$$

for the hand-to-mouth household. The first-order conditions of a saver at the non-stochastic steady-state are

$$\tilde{W} = \varphi^{\mathcal{S}} L^{\psi} (C^{\mathcal{S}})^{\gamma}, \tag{45}$$

and

$$\frac{R}{\Pi} = \frac{1}{\beta}. \tag{46}$$

The optimality condition for relative consumption for both types of household types is

$$\hat{C}_{ij}^{h} = \hat{P}_i^{-\theta}. \tag{47}$$

Lastly, firm i's first order condition is

$$\hat{P}_i = \frac{\tilde{W}}{\alpha} (\hat{P}_i^{-\theta} C)^{\frac{1-\alpha}{\alpha}}. \tag{48}$$

Equation (48) implies that all firms set the same price. Equation (47) then implies that each household consumes the same relative quantity of each consumption variety. Thus, all firms produce the same output, and since all firms have the same productivity, they also have the same labor input. Aggregate labor determines aggregate output, which in turn determines the market-clearing real wage and aggregate dividends. Equation (43), Equation (44) and Equation (45) determine the consumption levels.

The Euler equation of optimizing households, Equation (46), determines the real interest rate, but not R and Π individually. Thus, we will assume a constant price level such that $\Pi = 1$ and $R = \beta^{-1}$ and posit an initial value, P_{-1}, for the price level.

Given initial values for nominal bonds held by savers, $B_{-1} = B_{j,-1}\ \forall j \in [\phi, 1]$, fiscal variables are uniquely determined in the non-stochastic steady-state. The reason is that real bond holdings, B_{-1}/P_{-1}, is a quantity that must remain constant, and this can only be the case if the government runs a balanced budget in real terms. Thus, real lump-sum taxes must equate the sum of real interest and subsidy payments.

Given values for L, B and $\tilde{T}^{\mathcal{H}}$, all non-stochastic steady-state variables can be computed. Appendix C combines some of these relationships into expressions used in the approximation of the firms and households' objectives around that point.

B.3. Linearized Equilibrium We solve the perfect information numerically by linearizing its equilibrium conditions around its non-stochastic steady-state. Given that idiosyncratic technology shocks sum to zero in the cross-section, we drop indexes i for firms. Given that households of type h are ex-ante and ex-post identical, we also drop the indexes j. Equations (49) to (59) are the economy's equilibrium conditions

$$c_t^{\mathcal{S}} = E_t[c_{t+1}^{\mathcal{S}}] - \frac{1}{\gamma} E_t[r_t - \pi_{t+1}] \tag{49}$$

$$\tilde{w}_t = \psi l_t^{\mathcal{S}} + \gamma c_t^{\mathcal{S}} \tag{50}$$

$$\tilde{w}_t = \psi l_t^{\mathcal{H}} + \gamma c_t^{\mathcal{H}} \tag{51}$$

$$c_t^{\mathcal{H}} = \omega_w^{\mathcal{H}} (\tilde{w}_t + l_t^{\mathcal{H}}) \tag{52}$$

$$\tilde{w}_t = \tfrac{\alpha-1}{\alpha} y_t + \frac{1}{\alpha} a_t \tag{53}$$

$$r_t = \rho_r r_{t-1} + (1 - \phi_r)(\phi_\pi \pi_t) + v_t \tag{54}$$

$$c_t = \phi(C^{\mathcal{H}}/C)c_t^{\mathcal{H}} + (1-\phi)(C^{\mathcal{S}}/C)c_t^{\mathcal{S}} \tag{55}$$

$$l_t = \phi(L^{\mathcal{H}}/L)l_t^{h} + (1-\phi)(L^{\mathcal{S}}/L)l_t^{\mathcal{S}} \tag{56}$$

$$y_t = a_t + \alpha l_t \tag{57}$$

$$y_t = c_t \tag{58}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a. \tag{59}$$

Equation (49) and Equation (50) are the savers Euler equation and labor supply conditions. Equation (51) and Equation (52) are the hand-to-mouth's labor supply conditon and budget constraint. Equation (53) is the firms' optimal pricing condition. Equation (54) is the Taylor rule. Equation (55) and Equation (56) defined aggregate consumption and labor. Equation (57) is aggregate output and Equation (58) is the equilibrium condition on the goods market. Equation (59) is the exogenous process for technology.

Definition 3. *Given an initial value and the following non-explosive sequence for the savers' real bond holdings*

$$\lim_{s\to\infty} E_t[\beta^{s+1}(\tilde{b}_{t+s+1} - \tilde{b}_{t+s})] = 0, \tag{60}$$

an equilibrium of the perfect information economy is a solution to the system of equations that collects Equations (49) to (59).

Appendix C. Non-Stochastic Steady-State

The following non-stochastic steady-state relationships, which hold for all economies analyzed, are useful for approximating the objective functions of households and firms (i.e. sum of expected dividends and utilities).

The combination of firm i's optimal pricing condition and production function yields

$$
\begin{aligned}
\hat{P}_i &= \tilde{W}\frac{1}{\alpha}(\hat{P}_i^{-\theta}C_i^{\frac{1}{\alpha}-1}) \\
\hat{P}_i &= \tilde{W}\frac{C_i^{\frac{1}{\alpha}}}{\alpha C_i} \\
\hat{P}_i &= \tilde{W}\frac{L_i}{\alpha C_i} \\
\alpha C &= \tilde{W}L.
\end{aligned} \tag{61}
$$

We used the fact that firms are symmetrical in the non-stochastic steady state to eliminate the indexes i in the last line.

Rearranging the labor supply condition for both household types yields

$$
\begin{aligned}
\varphi^h L_j^{\psi} &= (C_j^h)^{-\gamma}\tilde{W} \\
\varphi^h L_j^{1+\psi} &= (C_j^h)^{-\gamma}\tilde{W}L_j \\
\varphi^h L_j^{1+\psi} &= (C_j^h)^{1-\gamma}\frac{\tilde{W}L_j}{C_j^h} \\
\varphi^h L^{1+\psi} &= (C^h)^{1-\gamma}\omega_w^h.
\end{aligned} \tag{62}
$$

Again, we used the symmetry between households of the same type h to eliminate the indexes j in the last line.

Appendix D. Expected Losses from Suboptimal Actions

D.1. Firms First, we guess that model-implied demand for consumption variety i is[22]

$$
C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\tilde{\theta}} C_t. \tag{63}
$$

Second, we substitute the demand function into the expression for period nominal profits

$$
D_{it} = (1+\tau_p)P_{it}\hat{P}_{it}^{-\tilde{\theta}}C_t - W_t L_{it} \tag{64}
$$

[22] We can prove that optimal attention allocation yields a demand function of that form.

Third, we replace the labor input using the production function

$$D_{it} = (1+\tau_p)P_{it}\hat{P}_{it}^{-\tilde{\theta}}C_t - W_t\left(\frac{\hat{P}_{it}^{-\theta}C_t}{e^{a_t}e^{a_{it}}}\right)^{\frac{1}{\alpha}} \tag{65}$$

Next, we assume that, in period -1, households who own the firms value profits at time t using the following discount factor

$$Q_{-1,t} = \beta^t \Lambda(\{C_{jt}\}_{j\in[\phi,1]})\frac{1}{P_t} \tag{66}$$

where the functional $\Lambda(\cdot)$ is twice continuously differentiable and satisfies

$$\Lambda(\{C_{jt}^{\mathcal{S}}\}_{j\in[\phi,1]}) = (C^{\mathcal{S}})^{-\gamma} \tag{67}$$

in the non-stochastic steady-state.

We multiply Equation (65), the period nominal profits, by the stochastic discount factor (exempt of β^t) which yields

$$\Lambda(\{C_{jt}^{\mathcal{S}}\}_{j\in[\phi,1]})\left[(1+\tau_p)\hat{P}_{it}^{1-\theta}C_t - \left(\frac{\hat{P}_{it}^{-\theta}C_t}{e^{a_t}e^{a_{it}}}\right)^{\frac{1}{\alpha}}\tilde{W}_t\right]. \tag{68}$$

We denote Equation (68) the real period profits function.

We rewrite this expression in terms of log-deviations around the non-stocastic steady-state

$$\Lambda(\{C_{jt}^{\mathcal{S}}\}_{j\in[\phi,1]})\left[\frac{\theta}{\theta-1}Y\left(\int_0^1 e^{(1-\theta)\hat{p}_{it}+c_{jt}}\,dj\right) - \alpha Y e^{\frac{\theta}{\alpha}\hat{p}_{it}-\frac{1}{\alpha}(a_t+a_{it})+\tilde{w}_t}\left(\int_0^1 e^{(1-\theta)\hat{p}_{it}+c_{jt}}\,dj\right)\right] \tag{69}$$

Let $\boldsymbol{x}_t$ denote the variables appearing in firm i's real period profit function that the firm can affect and $\boldsymbol{\zeta}_t$ the vector of variables that are taken as given

$$\boldsymbol{x}_{it} = (\hat{p}_{it})' \tag{70}$$

$$\boldsymbol{\zeta}_{it} = \left(a_t, a_{it}, \tilde{w}_t, \{c_{jt}\}_{j\in[0,1]}\right)' \tag{71}$$

We define $\mathcal{F}_i$ as the functional obtained from multiplying the period real profit function, Equation (69), by β^t and summing over all t from zero to infinity.

We let $\tilde{\mathcal{F}}_i$ denote the second-order approximation of that functional around the non-stochastic steady-state

$$
\begin{aligned}
&E_{i,-1}\left[\tilde{\mathcal{F}}_i\left(\boldsymbol{x}_{i0}, \boldsymbol{\zeta}_{i0}, \boldsymbol{x}_{i1}, \boldsymbol{\zeta}_{i1}, \cdots\right)\right] \qquad (72)\\
\approx &\mathcal{F}_i\left(\mathbf{0}, \mathbf{0}, \cdots\right) + E_{i,-1}\left[\sum_{t=0}^{\infty} \beta^t \left(\boldsymbol{h}'_{\boldsymbol{x_i}}\boldsymbol{x}_{it} + \boldsymbol{h}'_{\boldsymbol{\zeta_i}}\boldsymbol{\zeta}_{it} + \frac{1}{2}\boldsymbol{x}_{it}{}'\boldsymbol{H}_{\boldsymbol{x}_i}\boldsymbol{x}_{it} + \boldsymbol{x}_{it}{}'\boldsymbol{H}_{\boldsymbol{x_i\zeta_i}}\boldsymbol{\zeta}_{it} + \frac{1}{2}\boldsymbol{\zeta}_{it}{}'\boldsymbol{H}_{\boldsymbol{\zeta}_i}\boldsymbol{\zeta}_{it}\right)\right]
\end{aligned}
$$

where the vectors $\boldsymbol{h}_{\boldsymbol{x}_i}$ and $\boldsymbol{h}_{\boldsymbol{\zeta}_i}$ are first derivatives with respect to $\boldsymbol{x}_{it}$ and $\boldsymbol{\zeta}_{it}$ evaluated at the non-stochastic steady-state respectively. Similarly, $\boldsymbol{H}_{\boldsymbol{x_i}}$, $\boldsymbol{H}_{\boldsymbol{\zeta_i}}$ and $\boldsymbol{H}_{\boldsymbol{x_i\zeta_i}}$ are matrices of second order derivatives evaluated at the non-stochastic steady-state.

We can show that under some regularity conditions Equation (72) converges to a finite element in $\mathcal{R}$ along with each of its components[23].

The process defining firm's i vector of optimal actions, noted $\boldsymbol{x}^*_{it}$, is defined by the following requirement

$$
\boldsymbol{h}_{\boldsymbol{x_i}} + \boldsymbol{H}_{\boldsymbol{x_i}}\boldsymbol{x}^*_{it} + \boldsymbol{H}_{\boldsymbol{x_i\zeta_i}}\boldsymbol{\zeta}_{it} = 0. \qquad (73)
$$

The requirement above implies the same equation obtained by log-linearizing the firms' optimal pricing condition.

Next, we define firm i's objective function as the losses incurred from suboptimal actions which reads

$$
\begin{aligned}
&E_{i,-1}\left[\tilde{\mathcal{F}}_i\left(\boldsymbol{x}_{i0}, \boldsymbol{\zeta}_{i0}, \boldsymbol{x}_{i1}, \boldsymbol{\zeta}_{i1}, \cdots\right)\right] - E_{i,-1}\left[\tilde{\mathcal{F}}_i\left(\boldsymbol{x}^*_{i0}, \boldsymbol{\zeta}_{i0}, \boldsymbol{x}^*_{i1}, \boldsymbol{\zeta}_{i1}, \cdots\right)\right] \qquad (74)\\
=&E_{i,-1}\left[\sum_{t=0}^{\infty} \beta^t \left(\boldsymbol{h}_{\boldsymbol{x_i}}(\boldsymbol{x}_{it} - \boldsymbol{x}^*_{it}) + \frac{1}{2}\boldsymbol{x}_{it}\boldsymbol{H}_{\boldsymbol{x_i}}\boldsymbol{x}_{it} - \frac{1}{2}\boldsymbol{x}^*_{it}\boldsymbol{H}_{\boldsymbol{x_i}}\boldsymbol{x}^*{}_{it} + (\boldsymbol{x}_{it} - \boldsymbol{x}^*_{it})\boldsymbol{H}_{\boldsymbol{x_i\zeta_i}}\boldsymbol{\zeta}_{it}\right)\right]
\end{aligned}
$$

Lastly, we use Equation (73) to substitute for the term $\boldsymbol{H}_{\boldsymbol{x_i\zeta_i}}\boldsymbol{\zeta}_{it}$ in Equation (74). After rearranging we obtain

$$
\begin{aligned}
&E_{i,-1}\left[\tilde{\mathcal{F}}_i\left(\boldsymbol{x}_{i0}, \boldsymbol{\zeta}_{i0}, \boldsymbol{x}_{i1}, \boldsymbol{\zeta}_{i1}, \cdots\right)\right] - E_{i-1}\left[\tilde{\mathcal{F}}_i\left(\boldsymbol{x}^*_{i0}, \boldsymbol{\zeta}_{i0}, \boldsymbol{x}^*_{i1}, \boldsymbol{\zeta}_{i1}, \cdots\right)\right]\\
=&\frac{1}{2}(\boldsymbol{x}_{it} - \boldsymbol{x}^*_{it})\boldsymbol{H}_{\boldsymbol{x_i}}(\boldsymbol{x}_{it} - \boldsymbol{x}^*_{it}) \qquad (75)
\end{aligned}
$$

[23] See Maćkowiak and Wiederholt (2015) for the formal proof. The same result holds for households' approximations of period utilities defined below, provided certain initial conditions are satisfied for the savers.

where

$$\boldsymbol{H}_{\boldsymbol{x}_i} = -(C^{\mathcal{S}})^{-\gamma} Y \left[\frac{\theta(\theta + \alpha(1-\theta))}{\alpha}\right] \tag{76}$$

and

$$\boldsymbol{x}_{it}^{*} = \left(p_t + \frac{\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}} c_t + \frac{1}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}} \tilde{w}_t - \frac{\frac{1}{\alpha}}{1+\frac{1-\alpha}{\alpha}\tilde{\theta}} (a_t + a_{it})\right)'. \tag{77}$$

Notice that we used $\hat{p}_{it} - \hat{p}_{it}^{*} = p_{it} - p_{it}^{*}$ so that firms choose p_{it}, their price in level, instead of $\hat{p}_{it}$, their relative price. The only matrix of second-order derivatives needed to formulate firm i's attention problem in Section 4.1 is $\boldsymbol{H}_{\boldsymbol{x}_i}$.

D.2. Hand-to-Mouth First, we substitute the consumption aggregator into the flow budget constraint to get

$$C_{jt}\left(\int_0^1 P_{it} \hat{C}_{ijt} di\right) = W_t L_{jt} - T^{\mathcal{H}}. \tag{78}$$

Second, we isolate composite consumption and divide both the numerator and denominator by the price index

$$C_{jt} = \frac{\tilde{W}_t L_{jt} - \tilde{T}^{\mathcal{H}}}{\int_{[0,1)} \hat{P}_{it} \hat{C}_{ijt} di + \hat{P}_{1t}\left(1 - \int_{[0,1)} \hat{C}_{ijt}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} di}. \tag{79}$$

Notice that we partially relax mathematical rigor by treating the integral as a finite sum and di as a weight[24].

Third, we substitute the expression for consumption in the period utility function

[24]Throughout this section, we use this approach to ensure there is always a free variable when working with equality conditions. Alternatively, we could assume a finite number of households and firms, as in Maćkowiak and Wiederholt (2015), but this would make the relationship between aggregate and individual variables dependent on the size of the economy.

$$
\begin{aligned}
U(C_{jt}, L_{jt}) = \frac{1}{1-\gamma} & \left(\frac{\tilde{W}_t L_{jt} - \tilde{T}^{\mathcal{H}}}{\int_{[0,1)} \hat{P}_{it} \hat{C}_{ijt} di + \hat{P}_{1t} \left(1 - \int_{[0,1)} \hat{C}_{ijt}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} di} \right)^{1-\gamma} \\
& - \frac{1}{1-\gamma} \\
& - \varphi^{\mathcal{H}} \frac{L_{jt}^{1+\psi}}{1+\psi}.
\end{aligned} \tag{80}
$$

Next, we rewrite this expression in terms of log-deviations around the non-stocastic steady-state

$$
\begin{aligned}
U(C_{jt}, L_{jt}) = \frac{(C^{\mathcal{H}})^{1-\gamma}}{1-\gamma} & \left(\frac{\omega_w^{\mathcal{H}} e^{\tilde{w}_t + l_{jt}} + \omega_t^{\mathcal{H}}}{\int_{[0,1)} e^{\hat{p}_{it} + \hat{c}_{ijt}} di + e^{\hat{p}_{1t}} \left(1 - \int_{[0,1)} e^{\frac{\theta-1}{\theta} \hat{c}_{ijt}} di \right)^{\frac{\theta}{\theta-1}} di} \right)^{1-\gamma} \\
& - \frac{1}{1-\gamma} \\
& - \frac{(C^{\mathcal{H}})^{1-\gamma}}{1+\psi} \omega_w^{\mathcal{H}} e^{(1+\psi) l_{jt}}
\end{aligned} \tag{81}
$$

where $\omega_w^{\mathcal{H}} = \frac{\tilde{W} L}{C^{\mathcal{H}}}$ and $\omega_t^{\mathcal{H}} = \frac{\tilde{T}^{\mathcal{H}}}{C^{\mathcal{H}}}$ are non-stochastic steady-state ratios.

Let $\boldsymbol{x}_t$ denote the variables appearing in the period utility function that the hand-to-mouth households can affect and $\boldsymbol{\zeta}_t$ the vector of variables that are taken as given

$$
\boldsymbol{x}_t = \left(l_{jt}, \{\hat{c}_{ijt}\}_{i \in [0,1)} \right)' \tag{82}
$$

$$
\boldsymbol{\zeta}_t = \left(\tilde{w}_t, \{\hat{p}_t(i)\}_{i \in [0,1]} \right)' \tag{83}
$$

We define $\mathcal{F}^{\mathcal{H}}$ as the functional resulting from multiplying the period utility function, Equation (81), by β^t and summing over all t from zero to infinity.

We let $\tilde{\mathcal{F}}^{\mathcal{H}}$ denote the second-order approximation of that functional around the non-stochastic steady-state

$$E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{H}}\left(\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1\cdots\right)\right] \tag{84}$$

$$=\mathcal{F}^{\mathcal{H}}\left(\mathbf{0},\mathbf{0},\cdots\right)+E_{j,-1}\left[\sum_{t=0}^{\infty}\beta^t\left(\boldsymbol{h}_{\boldsymbol{x}}'\boldsymbol{x}_t+\boldsymbol{h}_{\boldsymbol{\zeta}}'\boldsymbol{\zeta}_t+\frac{1}{2}\boldsymbol{x}_t{}'\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_t+\boldsymbol{x}_t{}'\boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t+\frac{1}{2}\boldsymbol{\zeta}_t{}'\boldsymbol{H}_{\boldsymbol{\zeta}}\boldsymbol{\zeta}_t\right)\right] \tag{85}$$

where the vectors $\boldsymbol{h}_{\boldsymbol{x}}$ and $\boldsymbol{h}_{\boldsymbol{\zeta}}$ are first derivatives with respect to $\boldsymbol{x}_t$ and $\boldsymbol{\zeta}_t$ evaluated at the non-stochastic steady-state respectively. Similarly, $\boldsymbol{H}_{\boldsymbol{x}}$, $\boldsymbol{H}_{\boldsymbol{\zeta}}$ and $\boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}$ are the matrices of second order derivatives evaluated at the non-stochastic steady-state.

Under some regularity conditions Equation (84) and each of its elements converge to a finite element in $\mathcal{R}$.

The process defining the vector of optimal actions for household j, noted $\boldsymbol{x}_t^*$, is defined by the following requirement

$$\boldsymbol{h}_{\boldsymbol{x}}+\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_t^*+\boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t=0. \tag{86}$$

Next, we defined household j's objective function as the losses incurred from suboptimal actions which reads

$$E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{H}}\left(\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1,\cdots\right)\right]-E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{H}}\left(\boldsymbol{x}_0^*,\boldsymbol{\zeta}_0,\boldsymbol{x}_1^*,\boldsymbol{\zeta}_1,\cdots\right)\right] \tag{87}$$

$$=E_{j,-1}\left[\sum_{t=0}^{\infty}\beta^t\left(\boldsymbol{h}_{\boldsymbol{x}}(\boldsymbol{x}_t-\boldsymbol{x}_t^*)+\frac{1}{2}\boldsymbol{x}_t\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_t-\frac{1}{2}\boldsymbol{x}_t^*\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_t^*+(\boldsymbol{x}_t-\boldsymbol{x}_t^*)\boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t\right)\right]$$

Lastly, we use Equation (86) to substitute for $\boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t$ in Equation (87). After rearranging we obtain

$$E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{H}}\left(\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1,\cdots\right)\right]-E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{H}}\left(\boldsymbol{x}_0^*,\boldsymbol{\zeta}_0,\boldsymbol{x}_1^*,\boldsymbol{\zeta}_1,\cdots\right)\right] \tag{88}$$

$$=E_{j,-1}\sum_{t=0}^{\infty}\frac{1}{2}(\boldsymbol{x}_t-\boldsymbol{x}_t^*)\boldsymbol{H}_{\boldsymbol{x}}(\boldsymbol{x}_t-\boldsymbol{x}_t^*)$$

where

$$\boldsymbol{H}_x=-(C^{\mathcal{H}})^{1-\gamma}\begin{bmatrix}\omega_w^{\mathcal{H}}(\omega_w^{\mathcal{H}}\gamma+\psi) & 0 & \cdots & 0\\ 0 & \frac{2}{\theta}di & \cdots & \frac{1}{\theta}di\\ \vdots & \vdots & \ddots & \vdots\\ 0 & \frac{1}{\theta}di & \cdots & \frac{2}{\theta}di\end{bmatrix}, \tag{89}$$

$$
\boldsymbol{x}_t^* = \begin{pmatrix} \frac{\tilde{w}_t - \gamma c_{jt}^*}{\psi} \\ -\theta(p_{it} - p_t) \\ \vdots \end{pmatrix}, \tag{90}
$$

and

$$
c_{jt}^* = \omega_w^{\mathcal{H}}(\tilde{w}_t + l_{jt}^*) \tag{91}
$$

The only matrix of second-order derivatives needed to formulate the household j's attention problem in Section 4.2 is $\boldsymbol{H}_{\boldsymbol{x}}$ given by Equation (89).

D.3. Savers The first few steps are identical to those employed in the derivation of the hand-to-mouth's objective, except that in the period budget constraint other variables than labor income and taxes appear. In particular, bonds from period $t-1$ are a variable that saver get to choose.

We start by expressing consumption as a function of real variables and substitute in the period utility function. We get

$$
\begin{aligned}
U(C_{jt}, L_{jt}) = {} & \frac{1}{1-\gamma} \left(\frac{\frac{R_{t-1}}{\Pi_t}\tilde{B}_{jt-1} - \tilde{B}_{jt} + \tilde{W}_t L_{jt} + \tilde{D}_t^{\mathcal{S}} - \tilde{T}_t^{\mathcal{S}}}{\int_{[0,1)} \hat{P}_{it}\hat{C}_{ijt} di + \hat{P}_{1t}\left(1 - \int_{[0,1)} \hat{C}_{ijt}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} di} \right)^{1-\gamma} \\
& - \frac{1}{1-\gamma} - \varphi^{\mathcal{S}} \frac{L_{jt}^{1+\psi}}{1+\psi}.
\end{aligned} \tag{92}
$$

Next, we rewrite the period utility expression in terms of log-deviations from the non-stochastic steady-state

$$
\begin{aligned}
U(C_{jt}, L_{jt}) = {} & \frac{(C^{\mathcal{S}})^{1-\gamma}}{1-\gamma} \left(\frac{\frac{\omega_b^{\mathcal{S}}}{\beta} e^{r_{t-1} - \pi_t + \tilde{b}_{jt-1}} - \omega_b^{\mathcal{S}} e^{\tilde{b}_{jt}} + \omega_w^{\mathcal{S}} e^{\tilde{w}_t + l_{jt}} + \omega_d^{\mathcal{S}} e^{\tilde{d}_t^{\mathcal{S}}} - \omega_t^{\mathcal{S}} e^{\tilde{t}_t^{\mathcal{S}}}}{\int_{[0,1)} e^{\hat{p}_{it} + \hat{c}_{ijt}} di + e^{\hat{p}_{1t}}\left(1 - \int_{[0,1)} e^{\frac{\theta-1}{\theta}\hat{c}_{ijt}} di\right)^{\frac{\theta}{\theta-1}} di} \right)^{1-\gamma} \\
& - \frac{1}{1-\gamma} \\
& - \frac{(C^{\mathcal{S}})^{1-\gamma}}{1+\psi} \omega_w^{\mathcal{S}} e^{(1+\psi) l_{jt}}
\end{aligned} \tag{93}
$$

where $\omega_b^{\mathcal{S}}, \omega_d^{\mathcal{S}}, \omega_w^{\mathcal{S}}$, $\omega_t^{\mathcal{S}}$ are the following non-stochastic steady-state ratios

$$\left(\omega_w^{\mathcal{S}}, \omega_b^{\mathcal{S}}, \omega_d^{\mathcal{S}}, \omega_t^{\mathcal{S}}\right) = \left(\frac{\tilde{W}L}{C^{\mathcal{S}}}, \frac{\tilde{B}^{\mathcal{S}}}{C^{\mathcal{S}}}, \frac{\tilde{D}^{\mathcal{S}}}{C^{\mathcal{S}}}, \frac{\tilde{T}^{\mathcal{S}}}{C^{\mathcal{S}}}\right). \tag{94}$$

Let $\boldsymbol{x}_t$ denote the variables appearing in the period utility function that saver j can affect, and let the vector $\boldsymbol{\zeta}_t$ denote the variables that are taken as given such that

$$\boldsymbol{x}_t = \left(\tilde{b}_{jt}, l_{jt}, \{\hat{c}_{ijt}\}_{i\in[0,1)}\right)', \tag{95}$$

and

$$\boldsymbol{\zeta}_t = \left(r_{t-1}, \pi_t, \tilde{w}_t, \tilde{d}_t^{\mathcal{S}}, \tilde{t}_t^{\mathcal{S}}, \{\hat{p}_{it}\}_{i\in[0,1]}\right)'. \tag{96}$$

Additionally, we define the vector $\boldsymbol{x}_{-1}$, of the same length as $\boldsymbol{x}_t$, containing the variable $\tilde{b}_{j,-1}$, the only variable not present in either $\boldsymbol{x}_t$ or $\boldsymbol{\zeta}_t$

$$\boldsymbol{x}_{-1} = \left(\tilde{b}_{j,-1}, 0, \cdots\right)'. \tag{97}$$

We define $\mathcal{F}^{\mathcal{S}}$ as the functional resulting from multiplying the period utility function, Equation (93), by β^t and summing over all t from zero to infinity.

Letting $\tilde{\mathcal{F}}^{\mathcal{S}}$ denote the second-order Taylor approximaton of this functional evaluated at the non-stochastic steady-state, we get

$$E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{S}}\left(\boldsymbol{x}_{-1}, \boldsymbol{x}_0, \boldsymbol{\zeta}_0, \boldsymbol{x}_1, \boldsymbol{\zeta}_1, \cdots\right)\right] \tag{98}$$
$$= \begin{bmatrix} \mathcal{F}^{\mathcal{S}}\left(\mathbf{0}, \mathbf{0}, \mathbf{0}, \cdots\right) \\ +\sum_{t=0}^{\infty} \beta^t \begin{pmatrix} \boldsymbol{h}_{\boldsymbol{x}}'\boldsymbol{x}_t + \boldsymbol{h}_{\zeta}'\boldsymbol{\zeta}_t \\ +\frac{1}{2}\boldsymbol{x}_t'\boldsymbol{H}_{\boldsymbol{x},-1}\boldsymbol{x}_{t-1} + \frac{1}{2}\boldsymbol{x}_t'\boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}_t + \frac{1}{2}\boldsymbol{x}_t'\boldsymbol{H}_{\boldsymbol{x},1}\boldsymbol{x}_{t+1} \\ +\frac{1}{2}\boldsymbol{x}_t'\boldsymbol{H}_{\boldsymbol{x}\zeta}\boldsymbol{\zeta}_t + \frac{1}{2}\boldsymbol{x}_t'\boldsymbol{H}_{\boldsymbol{x}\zeta,1}\boldsymbol{\zeta}_{t+1} \\ +\frac{1}{2}\boldsymbol{\zeta}_t'\boldsymbol{H}_{\zeta}\boldsymbol{\zeta}_t + \frac{1}{2}\boldsymbol{\zeta}_t'\boldsymbol{H}_{\zeta\boldsymbol{x},-1}\boldsymbol{x}_{t-1} + \frac{1}{2}\boldsymbol{\zeta}_t'\boldsymbol{H}_{\zeta\boldsymbol{x}}\boldsymbol{x}_t \end{pmatrix} \\ +\beta^{-1}\left(\boldsymbol{h}_{-1}'\boldsymbol{x}_{-1} + \frac{1}{2}\boldsymbol{x}_{-1}'\boldsymbol{H}_{-1}\boldsymbol{x}_{-1} + \frac{1}{2}\boldsymbol{x}_{-1}'\boldsymbol{H}_{-1\boldsymbol{x}}\boldsymbol{x}_0 + \frac{1}{2}\boldsymbol{x}_{-1}'\boldsymbol{H}_{-1\zeta}\boldsymbol{\zeta}_0\right) \end{bmatrix}$$

We can show that under regularity conditions, each of the elements in Equation (98) converge to finite elements in $\mathcal{R}$[25].

The process defining the vector of optimal actions for savers j, noted $\boldsymbol{x}_t^*$, is defined by the following requirement

[25]The formal proof can be found in the appendix of Maćkowiak and Wiederholt (2015).

$$E_{jt}[\boldsymbol{h}_{\boldsymbol{x}} + \boldsymbol{H}_{\boldsymbol{x},-1}\boldsymbol{x}^*_{t-1} + \boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}^*_t + \boldsymbol{H}_{\boldsymbol{x},1}\boldsymbol{x}^*_{t+1} + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta},1}\boldsymbol{\zeta}_{t+1}] = 0. \tag{99}$$

We can rearrange Equation (99) to obtain the following expression

$$\begin{aligned} &E_{jt}[(\boldsymbol{x}_t - \boldsymbol{x}^*_t)'(\boldsymbol{h}_{\boldsymbol{x}} + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta}}\boldsymbol{\zeta}_t + \boldsymbol{H}_{\boldsymbol{x}\boldsymbol{\zeta},1}\boldsymbol{\zeta}_{t+1})] \\ =& - E_{jt}[(\boldsymbol{x}_t - \boldsymbol{x}^*_t)'(\boldsymbol{H}_{\boldsymbol{x},-1}\boldsymbol{x}^*_{t-1} + \boldsymbol{H}_{\boldsymbol{x}}\boldsymbol{x}^*_t + \boldsymbol{H}_{\boldsymbol{x},1}\boldsymbol{x}^*_{t+1})]. \end{aligned} \tag{100}$$

Next, using Equation (100) along with the assumption that $\boldsymbol{x}^*_{-1} = \boldsymbol{x}_{-1}$, some rearrangement yields the following expression quantifying the losses household j incurs due to suboptimal actions

$$\begin{aligned} &E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{S}}\left(\boldsymbol{x}_0, \boldsymbol{\zeta}_0, \boldsymbol{x}_1, \boldsymbol{\zeta}_1, \cdots\right)\right] - E_{j,-1}\left[\tilde{\mathcal{F}}^{\mathcal{S}}\left(\boldsymbol{x}^*_0, \boldsymbol{\zeta}_0, \boldsymbol{x}^*_1, \boldsymbol{\zeta}_1, \cdots\right)\right] \\ =& E_{j,-1}\sum_{t=0}^{\infty}\left[\frac{1}{2}(\boldsymbol{x}_t - \boldsymbol{x}^*_t)\boldsymbol{H}_{\boldsymbol{x}}(\boldsymbol{x}_t - \boldsymbol{x}^*_t) + (\boldsymbol{x}_t - \boldsymbol{x}^*_t)\boldsymbol{H}_{\boldsymbol{x},1}(\boldsymbol{x}_{t+1} - \boldsymbol{x}^*_{t+1})\right] \end{aligned} \tag{101}$$

where

$$\boldsymbol{H}_{\boldsymbol{x}} = -(C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix} \gamma\omega_b^2(1+\frac{1}{\beta}) & -\gamma\omega_b\omega_w & 0 & \cdots & 0 \\ -\gamma\omega_b\omega_b & \omega_w(\omega_w\gamma + \psi) & 0 & \cdots & 0 \\ 0 & 0 & \frac{2}{\theta}di & \cdots & \frac{1}{\theta}di \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \frac{1}{\theta}di & \cdots & \frac{2}{\theta}di \end{bmatrix}, \tag{102}$$

$$\boldsymbol{H}_{\boldsymbol{x},1} = (C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix} \gamma\omega_b^2 & -\gamma\omega_b\omega_w & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}. \tag{103}$$

and

$$\boldsymbol{x}^*_t = \begin{pmatrix} \frac{1}{\beta}(r_{t-1} - \pi_t + \tilde{b}^*_{jt-1}) + \frac{\omega_w^{\mathcal{S}}}{\omega_b^{\mathcal{S}}}(\tilde{w}_t + l^*_{jt}) + \frac{\omega_d^{\mathcal{S}}}{\omega_b^{\mathcal{S}}}\tilde{d}_t^{\mathcal{S}} - \frac{\omega_t^{\mathcal{S}}}{\omega_b^{\mathcal{S}}}\tilde{t}_t^{\mathcal{S}} - \frac{1}{\omega_b^{\mathcal{S}}}c^*_{jt} \\ \frac{\tilde{w}_t - \gamma c^*_{jt}}{\psi} \\ -\theta(p_{it} - p_t) \\ \vdots \end{pmatrix} \tag{104}$$

with

$$c^*_{jt} = E_{jt}\left[-\frac{1}{\gamma}(r_t - \pi_{t+1}) + c^*_{jt+1}\right] \text{ and } \omega_b^{\mathcal{S}} > 0. \tag{105}$$

D.4. Change of Variable. Equation (101) does not have the standard form of the objective in a dynamic attention rational inattention problem because of the intertemporal interaction term. We therefore perform a change of variable that allows us to write down the savers attention problem as a pure tracking problem.

We specifically focus on the 2 by 2 upper left elements of $\boldsymbol{H_x}$ and $\boldsymbol{H}_{\boldsymbol{x},1}$. The remaining terms relative to cross-sectional efficiency are unaffected by the following manipulations.

Equation (101) then reads

$$E_{j,-1}\sum_{t=0}^{\infty}\left[\frac{1}{2}(\bar{\boldsymbol{x}}_t-\bar{\boldsymbol{x}}_t^*)\bar{\boldsymbol{H}}_{\boldsymbol{x}}(\bar{\boldsymbol{x}}_t-\bar{\boldsymbol{x}}_t^*)+(\bar{\boldsymbol{x}}_t-\bar{\boldsymbol{x}}_t^*)\bar{\boldsymbol{H}}_{\boldsymbol{x},1}(\bar{\boldsymbol{x}}_{t+1}-\bar{\boldsymbol{x}}_{t+1}^*)\right] \tag{106}$$

with

$$\bar{\boldsymbol{x}}_t=\left(\tilde{b}_{jt},l_{jt}\right)', \tag{107}$$

$$\bar{\boldsymbol{H}}_{\boldsymbol{x}}=-(C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix}\gamma\omega_b^2(1+\frac{1}{\beta}) & -\gamma\omega_b\omega_w\\ -\gamma\omega_b\omega_w & \omega_w(\omega_w\gamma+\psi)\end{bmatrix}, \tag{108}$$

and

$$\bar{\boldsymbol{H}}_{\boldsymbol{x},1}=(C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix}\gamma\omega_b^2 & -\gamma\omega_b\omega_w\\ 0 & 0\end{bmatrix}. \tag{109}$$

Substituing Equations (107) to (109) in Equation (106), we obtain

$$-(C^{\mathcal{S}})^{1-\gamma}E_{j,-1}\sum_{t=0}^{\infty}\beta^t\begin{bmatrix}-\frac{1}{2}\begin{pmatrix}\gamma\omega_b^2(1+\frac{1}{\beta})(\tilde{b}_{jt}-\tilde{b}_{jt}^*)^2\\ -2\gamma\omega_b\omega_w(\tilde{b}_{jt}-\tilde{b}_{jt}^*)(l_{jt}-\bar{l}_{jt}^*)\\ +\omega_w(\gamma\omega_w+\psi)(l_{jt}-l_{jt}^*)^2\end{pmatrix}\\ +\gamma\omega_b^2(\tilde{b}_{jt}-\tilde{b}_{jt}^*)(\tilde{b}_{jt+1}-\tilde{b}_{jt+1}^*)\\ -\gamma\omega_b\omega_w(\tilde{b}_{jt}-\tilde{b}_{jt}^*)(l_{jt+1}-l_{jt+1}^*)\end{bmatrix}. \tag{110}$$

Next, we substract the linearized budget constraints evaluated at $\bar{\boldsymbol{x}}_t$ and $\bar{\boldsymbol{x}}_t^*$, we get

$$\omega_b(\tilde{b}_{jt}-\tilde{b}_{jt}^*)=\frac{\omega_b}{\beta}(\tilde{b}_{jt-1}-\tilde{b}_{jt-1}^*)-(c_{jt}-c_{jt}^*)+\omega_w(l_{jt}-l_{jt}^*). \tag{111}$$

We define the right-hand-side of Equation (111) as a new variable, Δ_t, proportional to mistakes in real bond holdings

$$\Delta_t = \omega_b(\tilde{b}_{jt} - \tilde{b}^*_{jt}). \tag{112}$$

Morevover, we can decompose Δ_t into two components, one reflecting mistakes in consumption and another specific to errors in labor supply such that

$$\Delta^c_t = \frac{1}{\beta}\Delta^c_{t-1} - (c_{jt} - c^*_{jt}) \tag{113}$$

and

$$\Delta^l_t = \frac{1}{\beta}\Delta^l_{t-1} + \omega_w(l_{jt} - l^*_{jt}) \tag{114}$$

with $\Delta^c_{-1} = 0$ and $\Delta^l_{-1} = 0$. By assumptions, we also have $(\tilde{b}_{j,-1} - \tilde{b}^*_{j,-1}) = 0$. Therefore, mistakes in real bond holdings are given by

$$\Delta_t = \Delta^c_t + \Delta^l_t. \tag{115}$$

Substituing Equation (112) and Equation (115) in Equation (110) yields

$$-(C^{\mathcal{S}})1 - \gamma E_{j,-1}\sum_{t=0}^{\infty}\beta^t \begin{bmatrix} -\frac{1}{2}\begin{pmatrix} \gamma(1+\frac{1}{\beta})(\Delta^c_t + \Delta^l_t)^2 \\ -2\gamma\omega_w(\Delta^c_t + \Delta^l_t)(l_{jt} - l^*_{jt}) \\ +\gamma\omega_w^2(l_{jt} - l^*_{jt})^2 + \psi\omega_w(l_{jt} - l^*_{jt})^2 \end{pmatrix} \\ +\gamma(\Delta^c_t + \Delta^l_t)(\Delta^c_{t+1} + \Delta^l_{t+1}) \\ -\gamma\omega_w(\Delta^c_t + \Delta^l_t)(l_{jt+1} - l^*_{jt+1}) \end{bmatrix}. \tag{116}$$

Next, we use Equation (114) to substitute for the term $(l_{jt+1} - l^*_{jt+1})$, and the first term that features $(l_{jt} - l^*_{jt})$ in Equation (116), we obtain

$$-(C^{\mathcal{S}})^{1-\gamma}E_{j,-1}\sum_{t=0}^{\infty}\beta^t \begin{bmatrix} -\frac{1}{2}\begin{pmatrix} \gamma(1+\frac{1}{\beta})(\Delta^c_t + \Delta^l_t)^2 \\ -2\gamma(\Delta^c_t + \Delta^l_t)(\Delta^l_t - \frac{1}{\beta}\Delta^l_{t-1}) \\ +\gamma(\Delta^l_t - \frac{1}{\beta}\Delta^l_{t-1})^2 + \psi\omega_w(l_{jt} - l^*_{jt})^2 \end{pmatrix} \\ +\gamma(\Delta^c_t + \Delta^l_t)(\Delta^c_{t+1} + \Delta^l_{t+1}) \\ -\gamma(\Delta^c_t + \Delta^l_t)(\Delta^l_{t+1} - \frac{1}{\beta}\Delta^l_t) \end{bmatrix}. \tag{117}$$

Rearranging, we obtain

$$-(C^{\mathcal{S}})^{1-\gamma}E_{j,-1}\sum_{t=0}^{\infty}\beta^t\begin{bmatrix}-\frac{\gamma}{2}(1+\frac{1}{\beta})(\Delta_t^c)^2+\gamma\Delta_t^c\Delta_{t+1}^c\\+\gamma\left[\Delta_t^l\Delta_{t+1}^c-\frac{1}{\beta}\Delta_{t-1}^l\Delta_t^c\right]\\+\frac{\gamma}{2\beta}\left[(\Delta_t^l)^2-\frac{1}{\beta}(\Delta_{t-1}^l)^2\right]\\-\frac{\omega_w\psi}{2}(l_{jt}-l_{jt}^*)^2\end{bmatrix}. \tag{118}$$

We combine and rewrite the first two terms, substituting in Equation (118) yields

$$-(C^{\mathcal{S}})^{1-\gamma}E_{j,-1}\sum_{t=0}^{\infty}\beta^t\begin{bmatrix}-\frac{\gamma}{2}(c_{jt}-c_{jt}^*)^2\\+\frac{\gamma}{2\beta}\left[(\Delta_t^c)^2-\frac{1}{\beta}(\Delta_{t-1}^c)^2\right]\\+\frac{\gamma}{2\beta}\left[(\Delta_t^l)^2-\frac{1}{\beta}(\Delta_{t-1}^l)^2\right]\\+\gamma\left[\Delta_t^c(c_{jt+1}-c_{jt+1}^*)-\frac{1}{\beta}\Delta_{t-1}^c(c_{jt}-c_{jt}^*)\right]\\-\frac{\omega_w\psi}{2}(l_{jt}-l_{jt}^*)^2\end{bmatrix}. \tag{119}$$

Summing terms appearing in consecutive periods, and using $\lim_{T\to\infty}\beta^T E_{j,-1}\left[(\Delta_T^c)^2\right]$ $=\lim_{T\to\infty}\beta^T E_{j,-1}\left[(\Delta_T^l)^2\right]=\lim_{T\to\infty}\beta^T E_{j,-1}\left[\Delta_T^c\Delta_{T+1}^l\right]=\lim_{T\to\infty}\beta^T E_{j,-1}\left[\Delta_T^c(c_{jT+1}-c_{jT+1}^*)\right]$ together with $\Delta_{-1}^c=\Delta_{-1}^l=0$ yields

$$-(C^{\mathcal{S}})^{1-\gamma}E_{j,-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{\gamma}{2}(c_{jt}-c_{jt}^*)^2+\frac{\omega_w\psi}{2}(l_{jt}-l_{jt}^*)^2\right]. \tag{120}$$

Under matricial form Equation (120) writes

$$E_{j,-1}\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2}(\tilde{\boldsymbol{x}}_t-\tilde{\boldsymbol{x}}_t^*)'\tilde{\boldsymbol{H}}_{\boldsymbol{x}}(\tilde{\boldsymbol{x}}_t-\tilde{\boldsymbol{x}}_t^*)\right]. \tag{121}$$

where

$$\tilde{\boldsymbol{H}}_{\boldsymbol{x}}=-(C^{\mathcal{S}})^{1-\gamma}\begin{bmatrix}\gamma & 0\\ 0 & \omega_w\psi\end{bmatrix}, \tag{122}$$

$$\tilde{\boldsymbol{x}}_t=(c_{jt},l_{jt})', \tag{123}$$

and

$$\tilde{\boldsymbol{x}}_t^*=\begin{pmatrix}E_{jt}\left[-\frac{1}{\gamma}(r_t-\pi_{t+1})+c_{jt+1}^*\right]\\ \frac{\tilde{w}_t-\gamma c_{jt}^*}{\psi}\end{pmatrix} \tag{124}$$

with c_{jt}^* and c_{jt+1}^* defined accordingly to Equation (105).

Appendix E. Households with Labor Market Power

In this section, we describe the modifications to the economic environment and the attention problems that arise when we assume that households have some market power, allowing them to set wages for their differentiated labor services.

E.1. Economic Environment. Firms aggregate the differentiated labor services into a single productive input using a CES aggregator

$$L_{it} = \left(\int_0^1 L_{ijt}^{\frac{\eta-1}{\eta}} \, dj \right)^{\frac{\eta}{\eta-1}} \tag{125}$$

where η is the elasticity of substitution between labor types.

As a result, firm i's optimal demand for labor of type j is given by

$$L_{ijt} = \left(\frac{W_{jt}}{W_t} \right)^{-\eta} L_{it}. \tag{126}$$

Inattention by firms implies $\tilde{\eta} < \eta$. We assume an aggregate wage index of the form

$$1 = \int_0^1 d_w \left(\hat{W}_{jt} \right) dj \tag{127}$$

where d_w is some twice continuously differentiabe function.

Note that outside the non-stochastic steady state, the average wage rates (and thus labor supplied) set by the two household types will differ due to the marginal rates of substitution between consumption and leisure not being equalized. The linearized aggregate wage rate faced by firms is given by

$$\tilde{w}_t = \phi \tilde{w}_t^{\mathcal{H}} + (1-\phi) \tilde{w}_t^{\mathcal{S}}. \tag{128}$$

The non-stochastic steady state of this economy is identical to that described in Appendix B.2, except for the inclusion of Equation (126), which has no effect in a symmetric equilibrium. To ensure that the model's wage elasticity is consistent with the economic environment, we may assume that households are subject to idiosyncratic productivity shocks, which have no effect on the aggregate dynamics.

E.2. Expected Losses from to Suboptimal Actions. The firm's attention problem remains the same once we abstract from cross-sectional efficiency (i.e., labor mix decisions). The matrices in the period payoff functions and the optimal actions of the wage-setting households are defined below. As before, we present only the elements relevant to aggregate dynamics..

Hand-to-mouth. The matrix appearing in household j of type $\mathcal{H}$'s period payoff function is

$$\boldsymbol{H}^{\mathcal{H}}_{\boldsymbol{x_j}} = -(C^{\mathcal{H}})^{1-\gamma} \left[\tilde{\eta}\omega^{\mathcal{H}}_w(1+\tilde{\eta}(\gamma\omega^{\mathcal{H}}_w+\psi))\right]. \tag{129}$$

Household j of type $\mathcal{H}$'s vector of choice variables and optimal actions are respectively

$$\tilde{\boldsymbol{x}}_{jt} = \left(\tilde{w}_{jt}\right)' \tag{130}$$

and

$$\tilde{\boldsymbol{x}}^*_{jt} = \left(\tfrac{\gamma\omega^{\mathcal{H}}_w+\psi}{1+\tilde{\eta}\psi+\gamma\omega^{\mathcal{H}}_w(\tilde{\eta}-1)}\left[\psi\left(\tilde{\eta}\tilde{w}_t+l_t\right)\right]\right). \tag{131}$$

Savers. The matrix appearing in household j of type $\mathcal{S}$'s period payoff is

$$\boldsymbol{H}^{\mathcal{S}}_{\boldsymbol{x_j}} = -(C^{\mathcal{S}})^{1-\gamma} \begin{bmatrix} \gamma & 0 \\ 0 & \tilde{\eta}\omega^{\mathcal{S}}_w(1+\tilde{\eta}\psi) \end{bmatrix}. \tag{132}$$

Household j of type $\mathcal{S}$'s vector of choice variables and optimal actions are respectively

$$\boldsymbol{x}_{jt} = \begin{pmatrix} c_{jt}, & \tilde{w}_{jt} \end{pmatrix}' \tag{133}$$

and

$$\boldsymbol{x}^*_{jt} = \begin{pmatrix} E_{jt}\left[-\frac{1}{\gamma}(r_t-\pi_{t+1})+c^*_{jt+1}\right] \\ (1+\tilde{\eta}\psi)^{-1}\left[\gamma c^*_{jt}+\psi\left(\tilde{\eta}\tilde{w}_t+l_t\right)\right] \end{pmatrix} \tag{134}$$

with c^*_{jt} and c^*_{jt+1} defined in accordance with Equation (105).

Lastly, the equilibrium around the non-stochastic steady-state, as well as the numerical procedure used to solve for it, are analogous to those described in the main text for the economy with a perfectly competitive labor market.

Appendix F. Calibration

F.1. Estimating the Share of Hand-to-Mouth We estimate the share of hand-to-mouth in the economy using the SCF, a triennial survey conducted by the Federal Reserve Board that collects detailed information on household finances in the United

States. It covers a representative sample of over 4,500 households and provides data on income, assets and debts.

First, we define the conditions under which a household classifies as hand-to-mouth;

Definition 4. *A household is considered hand-to-mouth if it either has*

- *zero liquid wealth*
- *or its credit limit is binding.*

This definition is standard in the literature, and is derived from the endogenous behavior of households in hetereogenous agent incomplete market models. As shown in Kaplan and Violante (2014), a household at a kink in its budget constraint, either at zero liquid wealth or the credit limit, exhibits a strong propensity to consume, which typically translates into allocating all of its period income to consumption and debt repayments, but none to savings. Hand-to-mouth households in our simplified framework display the exact same behavior.

Kaplan, Violante, and Weidner (2014) propose identifying hand-to-mouth households in the data using the following inequality conditions;

1. $0 \leq m_t(i) \leq \frac{w_t(i)}{2f}$
2. $m_t(i) \geq 0$ and $m_t(i) \leq \frac{w_t(i)}{2f} - \underline{m}_t(i)$

where $m_t(i)$ represents net liquid wealth, $w_t(i)$ denotes monthly income, and $f \geq 1$ is the frequency at which a household receives payments during a month[26].The first inequality identifies households with zero liquid wealth, while the second identifies those at their credit limit.

We estimate monthly income by dividing reported annual income by 12. We measure net liquid wealth based on the composition of liquid assets and debt defined in Table 8. We follow Kaplan and Violante (2014) and inflate the value of transactional accounts (LIQ) by a factor of 1.05 to account for cash holdings that are not reported in the SCF. In the model, borrowing should be considered as unsecured credit, so debt is measured using revolving credit card debt (i.e., credit card balances that are not repaid in full at every payment) which avoids including as debt purchases made through credit cards in between regular payments (see e.g., Telyukova (2013)). We infer revolving credit card debt by multiplying credit card balances after the last

[26]Income is measured monthly because, when $f = 1$, it represents the lowest plausible payment frequency. For more details on the estimator, see Kaplan, Violante, and Weidner (2014).

payment (CCBAL) with an indicator function that takes the value one if respondents answered either "sometimes" or "almost never" to the question "How often do you pay your credit card balance in full?". To ensure consistency with our model, we exclude households with zero wage income, those with a respondent under 22 or over 79 years old, and those above the 95th income percentile.

Table 8: Assets and Debt in the SCF

Category	Mnemonic
Liquid Assets	
All types of transaction account	LIQ
Directly held pooled investment funds	NMMF
Directly held stocks	STOCKS
Liquid Debt	
Credit card balances after last payment	CCBAL

Notes: This is a conservative definition of liquid wealth, the same as in Kaplan and Violante (2014). One could extend it to include assets with a lesser degree of liquidity.

F.2. Consumption Ratio We use our estimates of hand-to-mouth households in Appendix F.1 to calibrate the non-stochastic steady-state consumption ratio. First, we measure wage income and net liquid assets within each group. For both variables, we take the median. Wage income is directly reported in the survey, while net liquid assets is computed for each household by taking the difference between liquid assets and debts defined in Table 8.

We follow the strategy of Maćkowiak and Wiederholt (2015) to estimate consumption expenditures. First, apply the tax rate schedule[27] to median total income within each group. Next, we compute the total savings required to maintain median liquid net worth constant at an annual inflation rate of 2.5%. Consumption expenditures for each group is obtained by subtracting median savings from after-tax median income, and the ratio follows directly.

Appendix G. New Keynesian Frictions

Throughout the paper, we also reference models in which some of the slow adjustment in prices or quantities are due to adjustement frictions instead of imperfect

[27]Specifically, we apply the tax rate for "married filing jointly". Tax schedule for a given year can be found at https://www.irs.gov/pub/irs-prior.

information. In this section, we provide an overview of the microfoundations behind the following frictions: (i) sticky prices and (ii) sticky wages *à la* Calvo, and (ii) habit formation in consumption.

G.1. Sticky Prices. With probability $1 - \zeta_p$, firms are free to reset their price P_{it}. Conditional on resetting, they choose the optimal price P_{it}^* that maximizes the discounted sum of expected dividends.

Optimal price-setting gives rise to a New Keynesian Phillips curve, which, when linearized, is

$$\pi_t = \kappa_p s_t + \beta E_t[\pi_{t+1}] \tag{135}$$

with

$$\kappa_p = \frac{(1-\zeta_p)(1-\beta\zeta_p)}{\zeta_p}\left(\frac{\alpha}{\alpha + (1-\alpha)\theta}\right) \tag{136}$$

and

$$s_t = \tilde{w}_t + \frac{1-\alpha}{\alpha} y_t - \frac{1}{\alpha} a_t. \tag{137}$$

Equation (135) is the slope of the Phillips curve and Equation (137) is the real marginal cost. A complete derivation can be found in Galí (2015).

G.2. Sticky Wages. We assume that every household supply a continuum of differentiated labor services with wage elasticity of demand η. The wages for all types of labor is set by a union that chooses it to maximize the average utility of its members, given the demand it faces and subject to Calvo frictions. For each of labor services, the union can reset the wage rate at which it remunerated with unconditional probability $1 - \zeta_w$.

We allow each household type to be represented by its own union. This structure gives rise to a New Keynesian Phillips curve for the wage index of each household type, of the form

$$\tilde{w}_t^h = \frac{1}{1+\beta}\tilde{w}_{t-1}^h + \frac{\beta}{1+\beta}E_t\tilde{w}_{t+1}^h + \kappa_w^h\left(mrs_t^h - \tilde{w}_t^h\right) + \frac{1}{1+\beta}\pi_t + \frac{\beta}{1+\beta}E_t\pi_{t+1} \tag{138}$$

with

$$\kappa_w^h = \frac{(1-\zeta_w)(1-\beta\zeta_w)}{\zeta_w(1+\psi\eta)} \tag{139}$$

and

$$mrs_t^h = \psi l_t^h + \gamma c_t^h. \tag{140}$$

Equation (139) and Equation (140) are respectively the slope of the New Keynesian Philips curve for wages and the marginal rate of substitution between labor supply and consumption for household of type h.

Lastly, a linearization of the wage index implies

$$\tilde{w}_t = \phi \tilde{w}_t^{\mathcal{H}} + (1-\phi)\tilde{w}_t^{\mathcal{S}}. \tag{141}$$

One can also assume that a single unions represent all households as in Debortoli and Galí (2024). In that case, the relevant marginal rate of subsitution is a weighted average of the savers and hand-to-mouth.

G.3. Habit Formation. We modify the savers preferences such that utility depends not only on today's consumption but also on its value in the previous period

$$U(C_{jt}, C_{jt-1}, L_{jt}) = \frac{(C_{jt} - hC_{jt-1})^{1-\gamma} - 1}{1-\gamma} - \varphi^{\mathcal{S}} \frac{L_{jt}^{1+\psi}}{1+\psi}. \tag{142}$$

where $h \in [0,1]$ denotes the degree of habit formation.

A linearization of the Euler equation resulting from utility maximization for optimizing households yields the following two equations,

$$\lambda_t^{\mathcal{S}} = E_t[\lambda_{t+1}^{\mathcal{S}} + r_t - \pi_{t+1}] \tag{143}$$

and

$$\lambda_t^{\mathcal{S}} = \frac{\gamma h}{(1-h)(1-\beta h)} c_{t-1}^{\mathcal{S}} - \frac{\gamma(1+\beta h^2)}{(1-h)(1-\beta h)} c_t^{\mathcal{S}} + \frac{\gamma \beta h}{(1-h)(1-\beta h)} E_t[c_{t+1}^{\mathcal{S}}]. \tag{144}$$

where $\lambda_t^{\mathcal{S}}$ is the Lagrange multiplier on the household's flow budget constraint.

G.4. Linearized Optimality Conditions. To include the adjustment frictions in the economic environment, summarized by Equations (49) to (59), we proceed as follows. For Calvo prices, replace the firms' optimal pricing equations with the New Keynesian Phillips curve, Equations (135) to (137). For Calvo wages, replace the households labor supply equations with the New Keynesian Phillips curve for wages with Equations (138) to (141). For habit formation in consumption, substitute the savers' Euler equation with Equations (143) to (144). Modify the Taylor rule to account for the presence of an output gap due to frictions such that

$$r_t = \phi_r r_{t-1} + (1 - \phi_r)[\phi_\pi \pi_t + \phi_{y^*}(y_t - y_t^*)] \tag{145}$$

where y_t^* represents the natural level of output (i.e., the level of economic activity that would prevail in the frictionless, perfect information economy).

Note that, while typically not done in the rational inattention literature, it is possible to assume that some decisions are subject to adjustment frictions and others to inattention.