Computational Economics

Lecture 9: HANK Model

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Overview

We study a Heterogeneous Agent New Keynesian (HANK) model, a small-scale version following Auclert, Bardóczy, Rognlie & Straub (2021).

The model combines three building blocks:

1. **Households**: Incomplete markets with idiosyncratic labor income risk (Aiyagari-style). Households choose savings a and labor n.

2. **Firms**: Competitive final goods assembler + monopolistic intermediates subject to *Rotemberg* (1982) price adjustment costs $\Rightarrow$ NKPC.

3. **Policy**: Taylor rule (monetary) + fiscal authority issuing a fixed supply of bonds B.

Main reference: Auclert, Bardóczy, Rognlie & Straub, *Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models*, Econometrica (2021).

Households

Households: Setup

A unit mass of ex-ante identical households differ ex-post by:

- Asset holdings $a \in [\underline{a}, \infty)$ with borrowing limit $\underline{a} = 0$
- Labor productivity s ("skill")

Skill process: s follows a time-invariant discrete Markov chain with $n_s = 7$ states and transition matrix Π_s, discretized from an AR(1) in logs:

$$\log s' = \rho_s \log s + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \sigma_s^2)$$

with $\rho_s = 0.966$. The stationary distribution π_s satisfies $\int s \, d\pi_s = 1$ (normalized).

Note: s also scales lump-sum taxes paid and dividends received, both distributed proportionally to skill.

Households: Optimization Problem

Household savings a' and labor n solve the Bellman equation:

$$V_t(a, s) = \max_{c,\, n,\, a' \geq 0} \; u(c, n) + \beta \, \mathbb{E} \left[V_{t+1}(a', s') \mid s \right]$$

subject to the budget constraint:

$$c + a' = (1 + r_t)\, a + w_t \cdot s \cdot n + T_t \cdot s$$

where r is the real interest rate, w the real wage, and T the net lump-sum transfer: dividends minus taxes $(d_t - \tau_t)$.

The period utility function is additively separable:

$$u(c, n) = \frac{c^{1-\sigma}}{1-\sigma} - \varphi \, \frac{n^{1+\nu}}{1+\nu}$$

Households: Optimality Conditions

The intratemporal optimality condition equates the marginal disutility of labor to the marginal utility of the income it generates:

$$\varphi\, n^{\nu} = w \cdot s \cdot c^{-\sigma} \quad \Longrightarrow \quad n = \left(\frac{w \cdot s \cdot c^{-\sigma}}{\varphi} \right)^{1/\nu}$$

The intertemporal Euler equation:

$$c^{-\sigma} = \beta(1 + r)\, \mathbb{E}\left[(c')^{-\sigma} \mid s \right]$$

Solution method: Endogenous Grid Method (EGM), modified for endogenous labor supply. For constrained households ($a' = 0$), consumption and labor are solved jointly by fixed-point iteration.

Households: Distribution

The stationary distribution $\mathcal{D}(a, s)$ over asset holdings and skill is obtained via the **Young (2010) histogram method**:

1. Given the savings policy $a'(a, s)$, build the transition matrix Λ mapping today's distribution to tomorrow's.
2. Solve for the stationary distribution as:

$$\mathcal{D} = \left(I - \Lambda'\right)^{-1} \text{ (first column, normalized)}$$

3. Aggregate variables are obtained by integrating policy functions against $\mathcal{D}$:

$$A = \int a' \, d\mathcal{D}, \qquad L = \int s \cdot n \, d\mathcal{D}$$

Firms

Firms: Structure

The production side has two layers:

- **Final goods firm** (competitive): assembles output using a CES aggregator

$$Y_t = \left(\int_0^1 y_{jt}^{1/\mu} \, dj \right)^{\mu}$$

giving rise to a standard CES demand system for intermediate goods. The intermediate firms' gross markup is μ.

- **Intermediate monopolists**: produce using only labor,

$$y_{jt} = Z_t \, l_{jt}$$

where Z_t is aggregate TFP. They are subject to quadratic price adjustment costs à la Rotemberg (1982).

Firms: New Keynesian Phillips Curve

In a symmetric equilibrium, gross inflation $\Pi_t = P_t/P_{t-1}$ evolves according to the Rotemberg NKPC:

$$\log(1+\pi_t) = \kappa\left(\frac{w_t}{Z_t} - \mu^{-1}\right) + E_t \frac{1}{1+r_{t+1}} \frac{Y_{t+1}}{Y_t} \log(1+\pi_{t+1})$$

where $\pi_t \equiv \Pi_t - 1$ is net inflation and κ is the NKPC slope.

Policy

Monetary Policy

The central bank sets the nominal interest rate according to a Taylor rule:

$$i_t = r^* + \theta_\pi \pi_t + \varepsilon_t^i$$

where r^* is the long-run natural rate and ε_t^i is a monetary policy shock.

- The real interest rate is determined by the Fisher equation:

$$r_t = i_{t-1} - \pi_t$$

i.e., the real rate in period t is determined by the *previously set* nominal rate and current inflation.

Fiscal Policy

The fiscal authority issues a constant supply of government bonds B each period.

- Lump-sum taxes τ_t are collected proportionally to skill s
- The fiscal authority runs a balanced budget:

$$\tau_t = r_t B_t$$

- Net lump-sum transfers to households:

$$T_t = \underbrace{Y_t - w_t L_t}_{\text{dividends}} - \underbrace{r_t B}_{\text{taxes}}$$

Note: Since s scales both taxes paid and transfers received, the fiscal block introduces *heterogeneous* effects of monetary policy shocks across the wealth and skill distribution — a channel absent in RANK models.

Equilibrium

Market Clearing

In equilibrium, the following three conditions must hold simultaneously:

- **Asset market**:

$$\int a' \, d\mathcal{D}(a, s) = B$$

- **Labor market**:

$$\int s \cdot n \, d\mathcal{D}(a, s) = L_t$$

- **Goods market** (Walras' Law):

$$\int c \, d\mathcal{D}(a, s) = Y_t$$

Note: Price adjustment costs vanish in the linearized model, so the goods market clears as $C = Y = ZL$ to a first-order approximation.

Solving the Model

Solving the Steady State

The steady state is solved by iterating between the household problem and market clearing:

1. **Normalize**: $Z = 1$, $\pi = 0 \Rightarrow w_{ss} = 1/\mu$, $Y_{ss} = L_{ss}$.
2. **Solve HH problem** via EGM given $(w_{ss}, r_{ss}, T_{ss}) \Rightarrow$ policy functions $c(a,s)$, $a'(a,s)$, $n(a,s)$.
3. **Build** transition matrix Λ, compute stationary distribution $\mathcal{D}_{ss}$.
4. **Update** parameters until endogenous steady state variables meets some targets (e.g., interest rate).

Computing Dynamics: Linearization

To compute impulse response functions (IRFs), we linearize the equilibrium conditions around the steady state.

The full model can be written as a system $F(\mathbf{X}, \mathbf{Z}) = 0$, where $\mathbf{X}$ collects the time paths of endogenous variables $\{\pi_t, Y_t, w_t, r_t, \ldots\}$ and $\mathbf{Z}$ the shock path. Linearizing around the steady state:

$$F_X \, d\mathbf{X} + F_Z \, d\mathbf{Z} = 0 \quad \Longrightarrow \quad d\mathbf{X} = -F_X^{-1} F_Z \, d\mathbf{Z}$$

The key insight is that for a *small* shock, the equilibrium response is approximately linear. All methods exploit this in some form:

- **MIT shocks**: simulate a single perfect-foresight transition path back to steady state. The response is read off directly — implicitly computing $-F_X^{-1} F_Z \, d\mathbf{Z}$ without ever forming F_X.
- **Reiter / SSJ**: explicitly compute F_X and solve the system. More expensive to set up, but gives the full linearized solution at once and is trivially reusable for any shock $d\mathbf{Z}$.

Summary

The small HANK model features:

- **Incomplete markets** with idiosyncratic risk $\Rightarrow$ non-degenerate wealth distribution and heterogeneous MPCs
- **Endogenous labor supply** interacted with the income process $\Rightarrow$ richer response to wage and rate shocks
- **NKPC** $\Rightarrow$ endogenous inflation dynamics
- **Fiscal-monetary interaction**: transfers scale with s $\Rightarrow$ distributional effects of monetary policy