

# Computational Economics

## Lecture 8: New Keynesian Model

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# Motivation: Limits of Neoclassical Models

In previous lectures, we studied **frictionless neoclassical economies**:

- Markets are perfectly competitive; prices are flexible and adjust instantaneously.
- Equilibria are (generically) **efficient** — the First Welfare Theorem applies.
- Money is **neutral**: changes in the nominal money supply have no real effects.

These properties make neoclassical models tractable and theoretically elegant, but they come at a cost: they struggle to match key features of the data.

# Motivation: What the Data Tells Us

Neoclassical models cannot match several well-documented empirical regularities:

- **Price stickiness:** Prices at the micro level adjust infrequently (Bils & Klenow 2004; Nakamura & Steinsson 2008).
- **Money non-neutrality:** Monetary policy shocks have significant, persistent effects on output and employment (Christiano, Eichenbaum & Evans 1999).
- **Slow quantity adjustment:** The dynamic responses of output and investment to macro shocks are hump-shaped.

# Overview

**New Keynesian (NK) models** provide a framework that addresses these failures by introducing nominal and real rigidities into microfounded rational expectations model.

In this lecture, we proceed in two steps:

1. **Baseline NK model:** Monopolistic competition + Calvo price stickiness  $\Rightarrow$  the *3-equation* NK model.  
(IS curve, NKPC, Taylor rule)
2. **Medium-scale NK model:** Add habit formation, sticky wages, and capital adjustment costs to generate realistic output and inflation persistence.

Main reference: Galì, *Monetary Policy, Inflation and the Business Cycle*.

# The Baseline New Keynesian Economy

# Firms

The primary departure from the neoclassical benchmark is twofold:

- **Monopolistic competition:** Firms sell differentiated goods. Optimal demand follows from a CES aggregator with price elasticity of substitution  $\theta$ , so each firm faces a downward-sloping demand curve.
- **Calvo price frictions:** In each period only a random fraction  $(1 - \zeta_p)$  of firms from a continuum  $[0, 1]$  are able to reset their prices; the remaining fraction  $\zeta_p$  keep their price unchanged.

Together, these features generate **price stickiness**: in response to a shock, only a fraction of firms can adjust, so the aggregate price level moves sluggishly.

## Firms: Optimal Price-Setting

A firm  $i \in [0, 1]$  that can reset its price at time  $t$  solves:

$$\max_{\{P_t^*\}} \sum_{\tau=0}^{\infty} \zeta_p^\tau E_t \{ \beta^\tau \Lambda_{t,t+\tau} [P_{it} Y_{it+\tau|t} - C(Y_{it+\tau|t})] \}$$

subject to

$$Y_{it+\tau|t} = \left( \frac{P_{it}}{P_{t+\tau}} \right)^{-\theta} Y_{t+\tau}$$

where  $\Lambda_{t,t+\tau} = (U_{c,t+\tau}/U_{c,t})\Pi_{t+\tau}^{-1}$  is a stochastic discount factor and  $C(\cdot)$  is the nominal cost function.

The firm discounts future profits by  $\beta^\tau \zeta_p^\tau$  because it expects to be stuck with today's price with probability  $\zeta_p^\tau$ .

**Note:** We implicitly assume that a firm can always meet demand at its current price, which requires markups to be large enough and demand shifts due to shocks to not be too large.

## Firms: Optimality Condition

Taking the derivative with respect to  $P_{it}$ , the firm's optimality condition is:

$$P_t^* = \frac{\theta}{\theta - 1} \frac{E_t \left[ \sum_{\tau=0}^{\infty} (\beta \zeta_p)^\tau Y_{t+\tau} P_{t+\tau}^{\theta-1} MC_{t+\tau|t} \right]}{E_t \left[ \sum_{\tau=0}^{\infty} (\beta \zeta_p)^\tau Y_{t+\tau} P_{t+\tau}^{\theta-1} \right]} \quad (1)$$

where  $MC_{t+\tau|t}$  is the nominal marginal cost of a firm that last reset its price in period  $t$ .

The firm sets its price as a **weighted average of current and future marginal costs**, since it may be unable to adjust in subsequent periods.

### Flexible price benchmark

Without frictions ( $\zeta_p = 0$ ), the firm sets a constant markup over current marginal cost:  $P_t^* = \frac{\theta}{\theta-1} MC_t$ .

## Firms: Price Level Dynamics

The CES demand function implies an aggregate price index:

$$P_t = \left( \int_0^1 P_{it}^{1-\theta} di \right)^{\frac{1}{1-\theta}}.$$

Given Calvo frictions (fraction  $\zeta_p$  of firms cannot adjust), this can be written recursively as:

$$P_t^{1-\theta} = (1 - \zeta_p)(P_t^*)^{1-\theta} + \zeta_p P_{t-1}^{1-\theta}. \quad (2)$$

This is the law of motion for the price level: it is a weighted average of the newly set price  $P_t^*$  and the lagged price level  $P_{t-1}$ , with weights determined by the fraction of adjusting firms.

**Note:** Price dispersion across firms generates **allocative inefficiencies** in the economy.

# The New Keynesian Phillips Curve

Taking a log-linear approximation around the non-stochastic steady state (nsss) of Equation (1) and Equation (2), combining and rearranging yields the **New Keynesian Phillips Curve (NKPC)**:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa_p s_t \quad (3)$$

where  $s_t$  is the (log-linearized) real marginal cost and  $\kappa_p \equiv \frac{(1-\zeta_p)(1-\zeta_p\beta)}{\zeta_p} \Theta$  is the slope of the NKPC (decreasing in  $\zeta_p$ ).

**Interpretation:** Current inflation is driven by (i) expected future inflation and (ii) real marginal cost — a measure of real activity. As  $\zeta_p \rightarrow 0$ , prices become flexible and the NKPC steepens.

# Households

Households can save in **nominal bonds** paying gross nominal return  $R_t$ . The first-order condition yields the Euler equation:

$$U_{c,t} = \beta E_t \left[ U_{c,t+1} \left( \frac{R_t}{\Pi_{t+1}} \right) \right] \quad (4)$$

where  $U_{c,t}$  is the marginal utility of consumption and  $\Pi_{t+1} = P_{t+1}/P_t$  is gross inflation. Log-linearizing around the nsss:

$$u_{c,t} = E_t [u_{c,t+1}] + E_t [r_t - \pi_{t+1}] \quad (5)$$

This is the **dynamic IS equation**: household consumption is a function of the real interest rate  $r_t - E_t[\pi_{t+1}]$  path.

# Central Bank

The nominal interest rate is set by the central bank according to a **Taylor rule**:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) \phi \pi_t \quad (6)$$

- $\rho_r \in [0, 1)$  is the interest rate smoothing parameter.
- $\phi > 1$  determines the central bank's response to inflation.

A condition, referred to as the **Taylor principle**, on  $\phi$  must be met for determinacy (i.e.,  $\phi$  must be large enough).

**Note:** The Taylor rule can be augmented to respond to other variables such as the output gap, output growth, etc.

## The 3-Equation NK Model

Assuming labor is the only input to production, Equation (3) and Equation (5) can be expressed as functions of the output gap  $\tilde{y}_t \equiv y_t - y_t^n$  (actual vs. natural output). Together with the Taylor rule Equation (6), the model closes as:

$$\begin{aligned}\tilde{y}_t &= E_t[\tilde{y}_{t+1}] - \sigma^{-1}(r_t - E_t[\pi_{t+1}] - r_t^n) \\ \pi_t &= \beta E_t[\pi_{t+1}] + \kappa \tilde{y}_t \\ r_t &= \rho_r r_{t-1} + (1 - \rho_r) \phi \pi_t\end{aligned}$$

This “3-equation” model is the workhorse for studying monetary policy transmission and the effects of nominal rigidities. (See Galì Ch. 3.)

# Medium Scale New Keynesian Economy

# Limitations of the Baseline Model

While tractable, the baseline 3-equation NK model struggles on several dimensions:

- **Lack of intrinsic persistence:** Inflation and output respond on impact but revert quickly — the model does not generate the *hump-shaped* impulse responses seen in the data.
- **No real rigidities:** Real marginal cost is too sensitive to output, making the NKPC slope  $\kappa$  implausibly large to match observed inflation dynamics.
- **No investment dynamics:** The baseline model has no capital, so it misses a key amplification and propagation channel.

⇒ Additional “patches” in the form of real and nominal rigidities are needed. This is the **medium-scale NK model** (Christiano, Eichenbaum & Evans 2005; Smets & Wouters 2007).

# Habit Formation

**Motivation:** Consumption exhibits **excess smoothness** and a hump-shaped response to shocks that the standard Euler equation cannot explain.

Households utility depend on current and previous period consumption:

$$U(C_t, C_{t-1}, N_t) = \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where  $h \in [0, 1)$  is the habit persistence parameter.

This specification makes households sensitive to the **growth rate** of consumption, introducing persistence in their response.

# Sticky Wages

**Motivation:** Wages are at least as sticky as prices in the data.

Analogously to price setting, we introduce **Calvo wage frictions**: in each period only a fraction  $(1 - \zeta_w)$  of households (who supply differentiated labor) can re-optimize their nominal wage.

This delivers a **New Keynesian Wage Phillips Curve**:

$$\pi_t^w = \beta E_t[\pi_{t+1}^w] + \kappa_w \mu_t^w$$

where  $\pi_t^w$  is wage inflation,  $\mu_t^w$  is the (log-deviation of the) wage markup, and  $\kappa_w \equiv \frac{(1-\zeta_w)(1-\zeta_w\beta)}{\zeta_w(1+\varphi\theta_w)}$ .

Wage stickiness dampens the response of real marginal cost to output fluctuations, flattening the price NKPC.

# Capital Adjustment Costs

**Motivation:** Investment is *much* more volatile than consumption in the data, yet also it also responds sluggishly to shocks.

We introduce **convex capital adjustment costs**: installed capital evolves as

$$K_{t+1} = \left[ 1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t + (1 - \delta)K_t$$

where  $S(\cdot)$  satisfies  $S(1) = S'(1) = 0$  and  $S''(1) \equiv \xi > 0$ .

These costs penalize *rapid changes* in investment, generating a **hump-shaped investment response**: firms smooth capital accumulation over time rather than adjusting on impact.

## Other Extensions

Sticky prices and wages, habit formation, and investment adjustment costs help bridge the gap between the model and the data. Medium-scale models are typically further enriched with:

- **Variable capital utilization:** Firms can vary the intensity of capital use, adding a margin of adjustment that amplifies output responses.
- **Wage and price indexation:** Non-optimizing firms and households index their prices and wages to lagged inflation, generating *intrinsic* inflation persistence even without re-optimization.
- **Multiple shocks:** Technology, preference, government spending, monetary policy, cost-push shocks, and more.

Beyond these, the framework can be extended to incorporate other aspects of the economy such as a financial sector.

# My Own Views

A few observations on the NK framework:

- Some of the adjustment frictions arguably lack solid microfoundations (i.e., the Calvo mechanism assumes a constant, history-independent probability of price adjustment).
- The medium-scale model **fits the data well by design**: with enough shocks and frictions, matching moments is not hard. Identification of structural parameters is challenging.

That said, the NK framework remains the **dominant paradigm** in central banking and policy analysis, and understanding its mechanics is worthwhile.